

Verifying Compiler Optimisation Passes

Brae Webb

Supervised by:
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Thanks to:
Kristian Thomassen
Ethan Bulmer
Kausthubram Rajesh

Roadmap

Roadmap

Motivation



Roadmap



Roadmap



Roadmap



Motivation

Verifying Compiler Optimization Passes

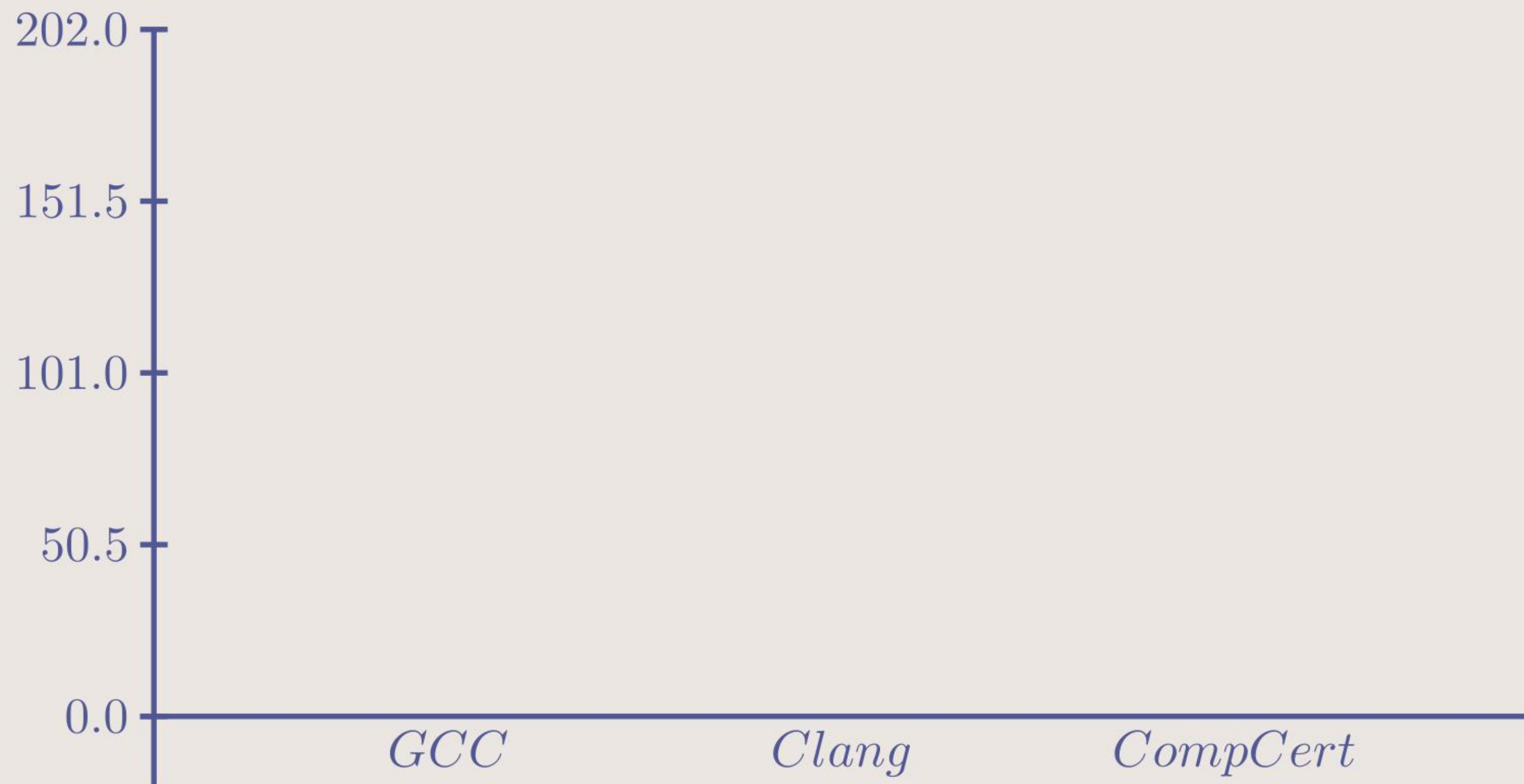
Verifying GraalVM Compiler Optimization Passes

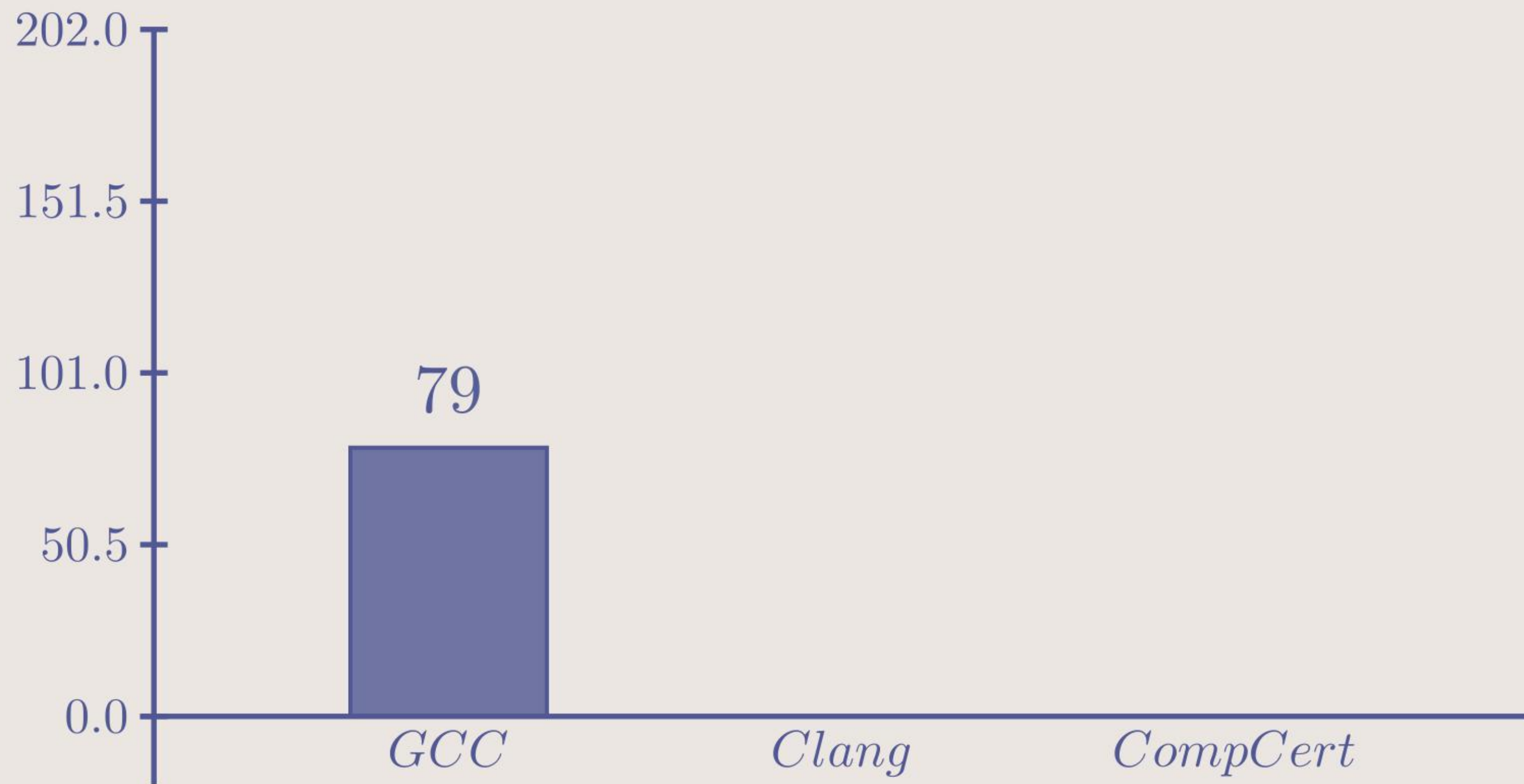
Verifying Compilers

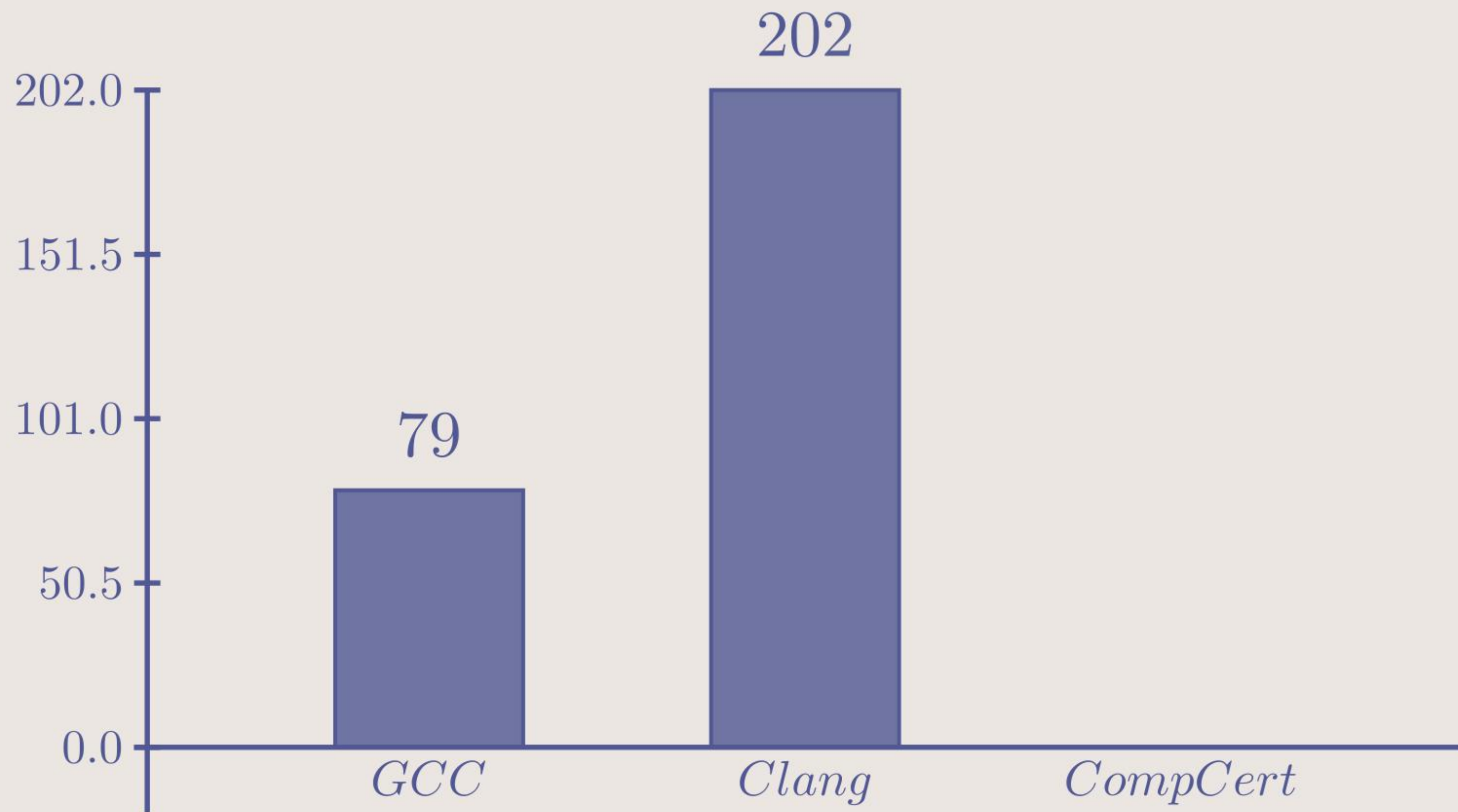
Finding and Understanding Bugs in C Compilers

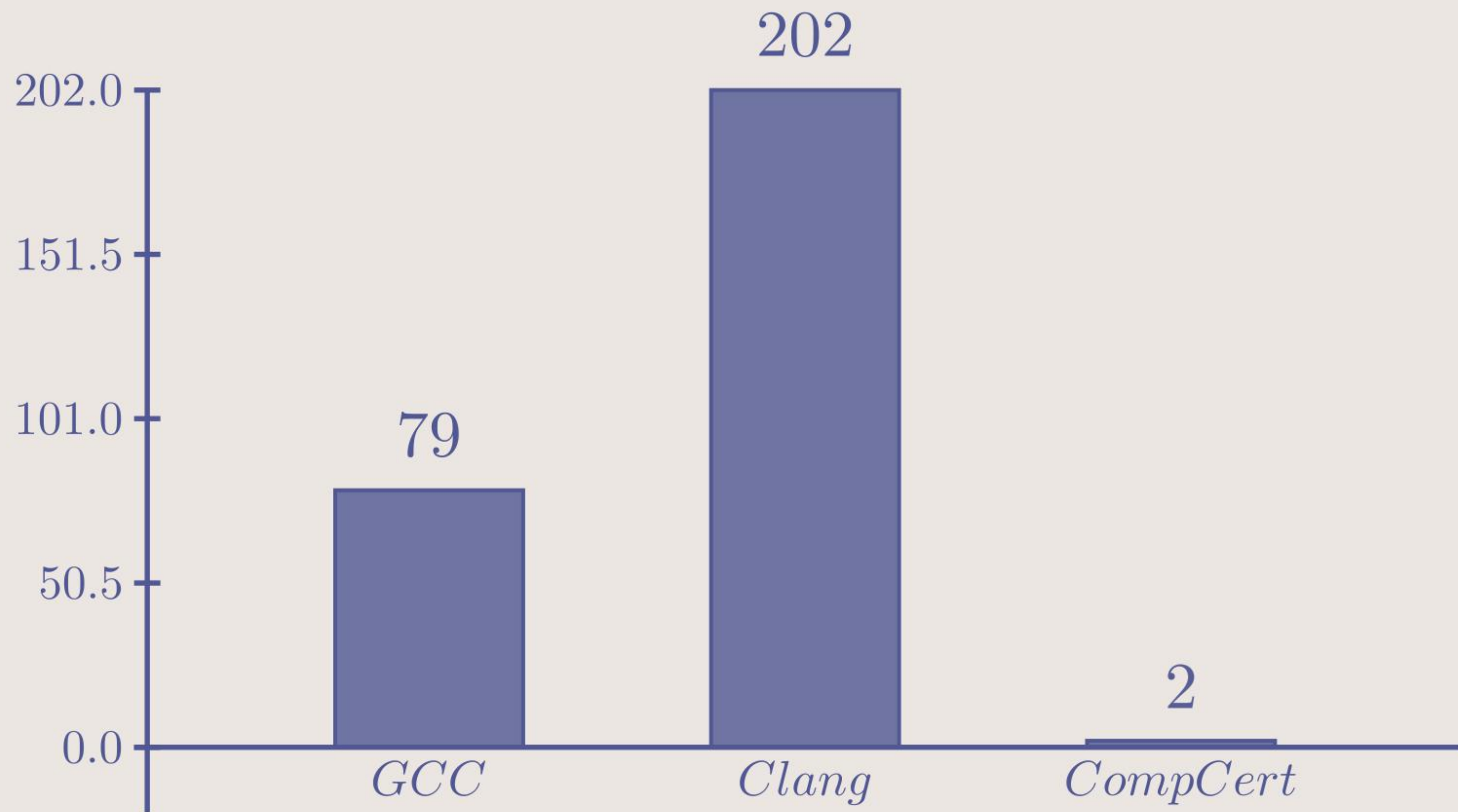
Xuejun Yang Yang Chen Eric Eide John Regehr

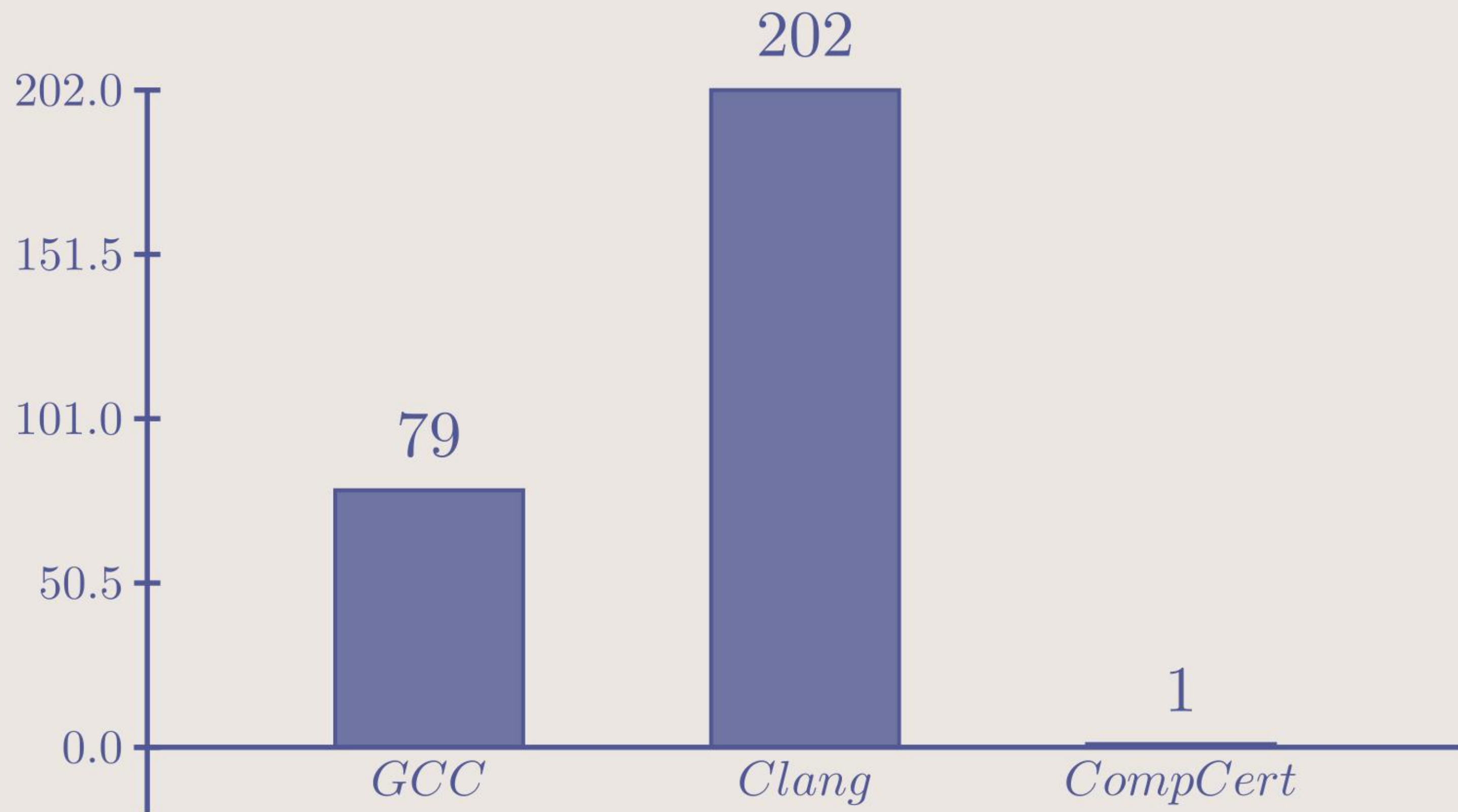
Conference on Programming Language
Design and Implementation (PLDI)
2011

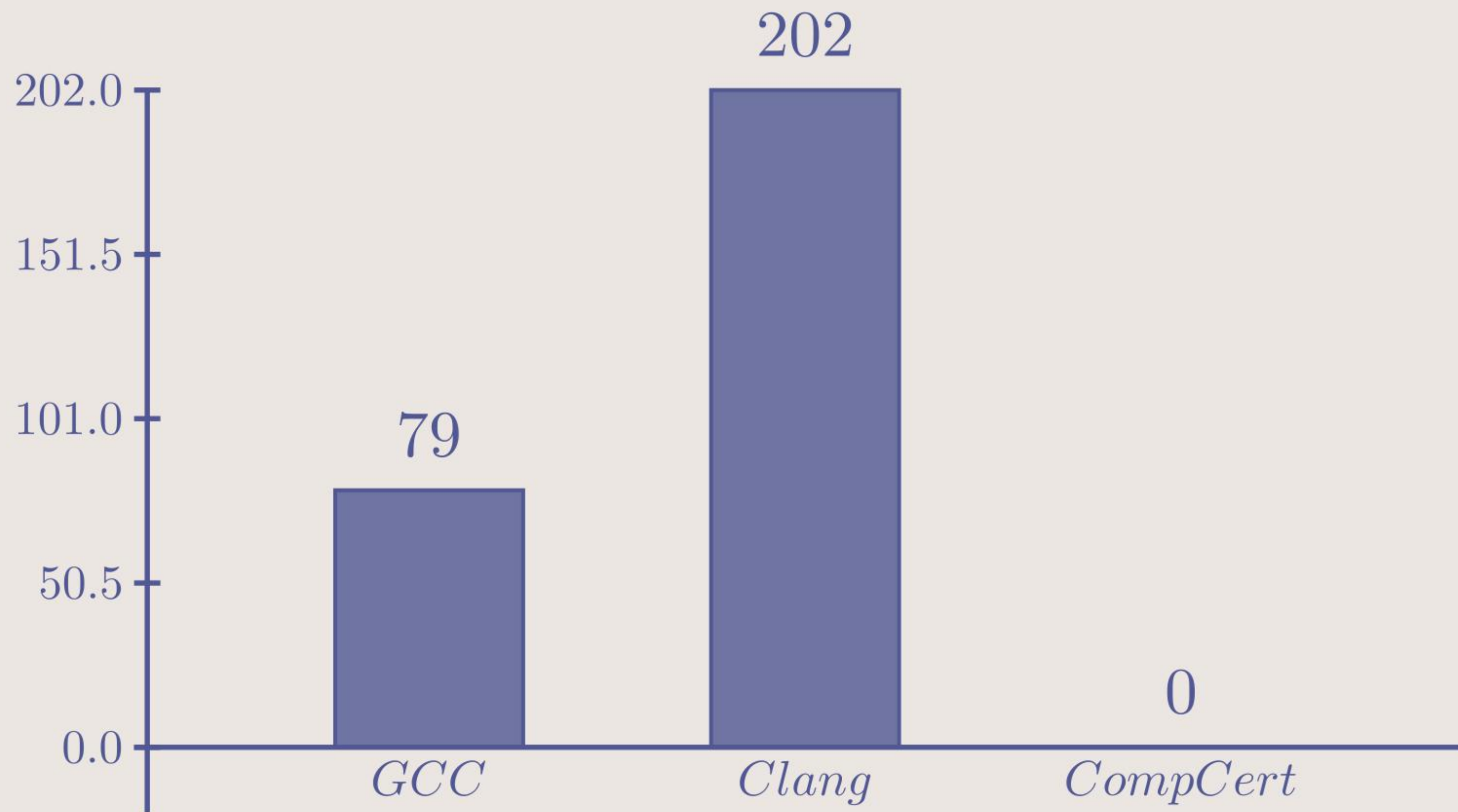






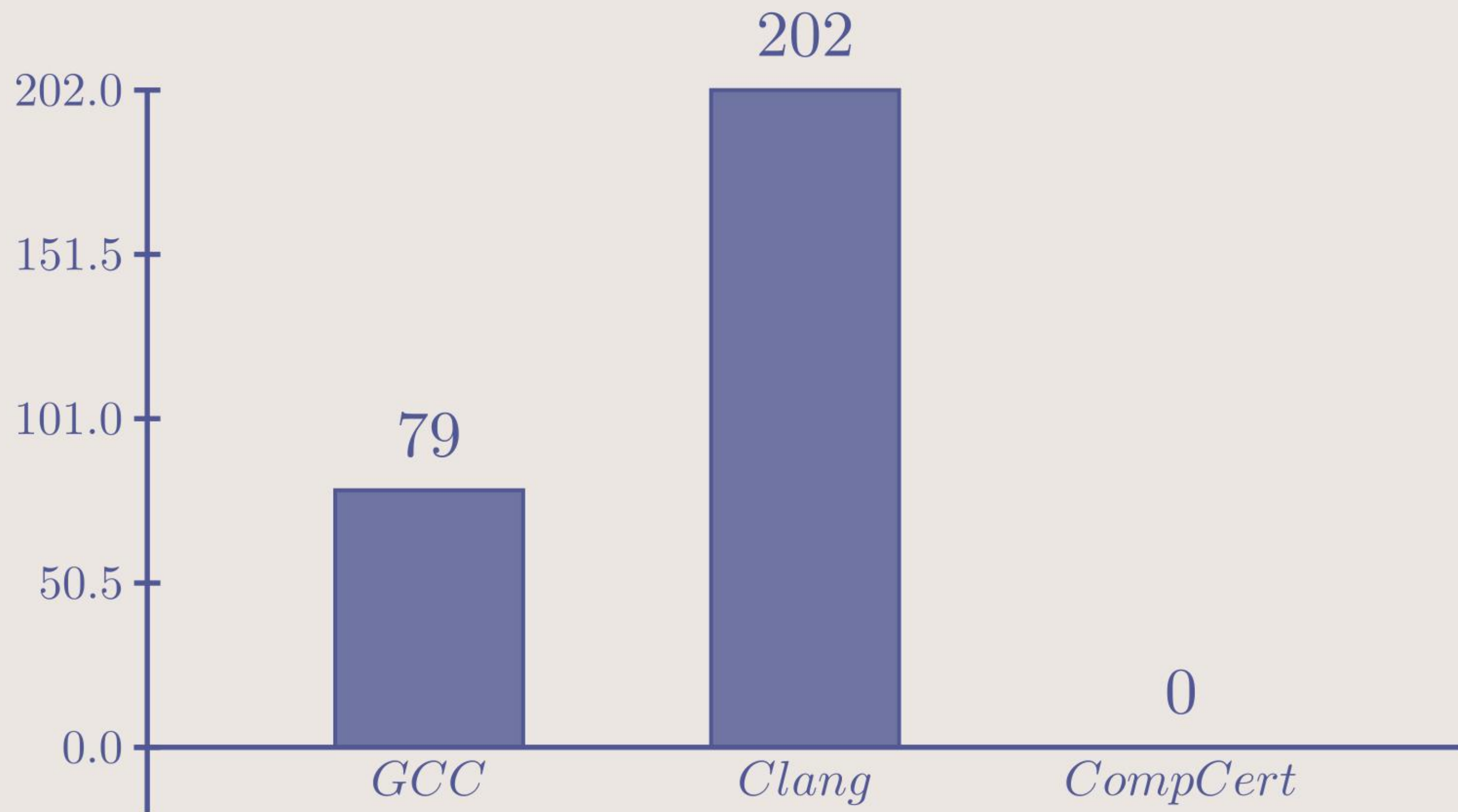


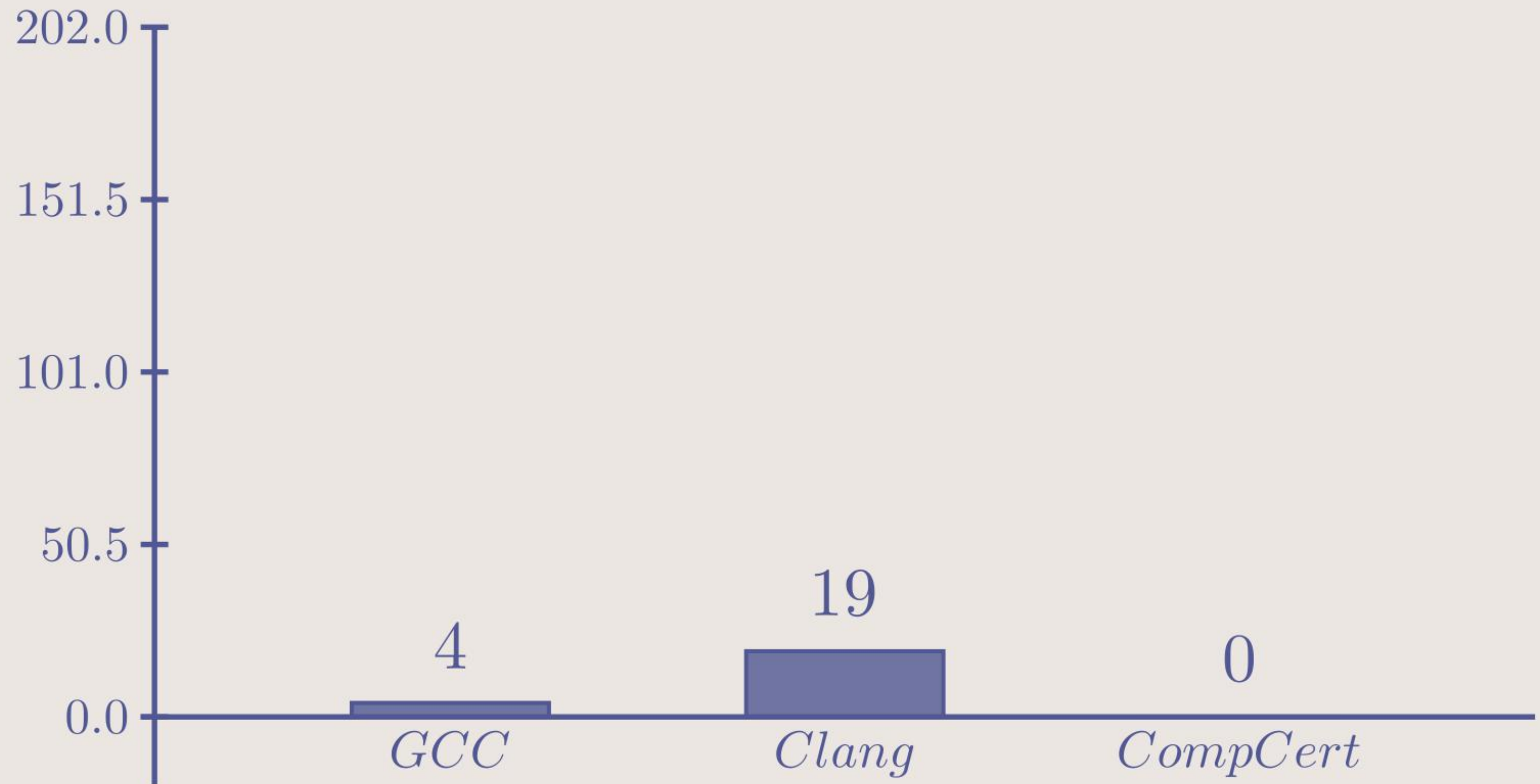




Verifying Compilers

Verifying Compiler Optimization Passes





Optimizing compilers are so difficult to get right
that we dare say that no optimizing compiler
is completely error-free!

“Optimizing compilers are so difficult to get right
that we dare say that no optimizing compiler
is completely error-free!”

- Aho, Lam, Sethi, Ullman in
Compilers: Principles, Techniques,
& Tools 2nd Edition

```
boolean isLessThan(Integer a, Integer b) {  
    if (a == b) {  
        return false;  
    }  
  
    if (a < b) {  
        return true;  
    }  
  
    if (a > b) {  
        return false;  
    }  
  
    return true;  
}
```

```
boolean isLessThan(Integer a, Integer b) {  
    if (a == b) {  
        return false;  
    }  
  
    if (a < b) {  
        return true;  
    }  
  
    if (a > b) {  
        return false;  
    }  
  
    return true;  
}
```

```
boolean isLessThan(Integer a, Integer b) {  
    return a < b;  
}
```

```
boolean isLessThan(Integer a, Integer b) {  
    if (a == b) {  
        return false;  
    }  
  
    if (a < b) {  
        return true;  
    }  
  
    if (a > b) {  
        return false;  
    }  
  
    return true;  
}
```

```
boolean isLessThan(Integer a, Integer b) {  
    return a < b;  
}
```

```
assert lessThan(0, 0) == lessThanOpt(0, 0)  
assert lessThan(0, 1) == lessThanOpt(0, 1)  
assert lessThan(1, 0) == lessThanOpt(1, 0)  
assert lessThan(-1, 100) == lessThanOpt(-1, 100)  
assert lessThan(50, 50) == lessThanOpt(50, 50)
```

```
boolean isLessThan(Integer a, Integer b) {  
    if (a == b) {  
        return false;  
    }  
  
    if (a < b) {  
        return true;  
    }  
  
    if (a > b) {  
        return false;  
    }  
  
    return true;  
}
```

```
boolean isLessThan(Integer a, Integer b) {  
    return a < b;  
}
```

```
assert lessThan(0, 0) == lessThanOpt(0, 0)  
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```

```
boolean isLessThan(Integer a, Integer b) {  
    if (a == b) {  
        return false;  
    }  
  
    if (a < b) {  
        return true;  
    }  
  
    if (a > b) {  
        return false;  
    }  
  
    return true;  
}
```

```
boolean isLessThan(Integer a, Integer b) {  
    return a < b;  
}
```

```
assert lessThan(0, 0) == lessThanOpt(0, 0)  
assert lessThan(0, 1) == lessThanOpt(0, 1)  
assert lessThan(1, 0) == lessThanOpt(1, 0)  
assert lessThan(-1, 100) == lessThanOpt(-1, 100)  
assert lessThan(50, 50) == lessThanOpt(50, 50)  
assert lessThan(2222, 2222) == lessThanOpt(2222, 2222)
```

Verifying Compiler Optimization Passes

Verifying GraalVM Compiler Optimization Passes

```
1  a + b - 0;  
2  a - c;
```

```
1  a + b - 0;  
2  a - c;
```

```
1  a + b - 0;  
2  a - c;
```

```
program = new Program(  
    SubNode(  
        AddNode(VariableNode('a'),  
                VariableNode('b')),  
        ConstantNode(0)  
    ),  
    SubNode(  
        VariableNode('a'),  
        VariableNode('c')  
    )  
)
```

```
1  a + b - 0;  
2  a - c;
```

```
program = new Program(  
    SubNode(  
        AddNode(VariableNode('a'),  
                VariableNode('b')),  
        ConstantNode(0)  
    ),  
    SubNode(  
        VariableNode('a'),  
        VariableNode('c')  
    )  
)
```

```
void executeProgram(Program program,  
                    SymTable symtab) {  
    for (line : program.getExpressions()) {  
        int value = line.execute(symtab);  
        print(value);  
    }  
}
```

```
1 a + b - 0;  
2 a - c;
```

```
program = new Program(  
    SubNode(  
        AddNode(VariableNode('a'),  
                VariableNode('b')),  
        ConstantNode(0)  
    ),  
    SubNode(  
        VariableNode('a'),  
        VariableNode('c')  
    )  
)
```

```
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        int value = line.execute(symtab);  
        print(value);  
    }  
}
```

```
class AddNode extends Expression {  
    Expression left;  
    Expression right;  
  
    int execute(SymTable symtab) {  
        return left.execute() + right.execute();  
    }  
}  
  
class ConstantNode extends Expression {  
    int value;  
  
    int execute(SymTable symtab) {  
        return value;  
    }  
}  
  
class VariableNode extends Expression {  
    String identifier;  
  
    int execute(SymTable symtab) {  
        symtab.lookup(identifier);  
    }  
}
```

```

program = new Program(
    SubNode(
        AddNode(VariableNode('a'),
            VariableNode('b')),
        ConstantNode(0)
    ),
    SubNode(
        VariableNode('a'),
        VariableNode('c')
    )
)

```

```

void executeProgram(Program program,
    SymTable symtab) {
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```

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class AddNode extends Expression {
    Expression left;
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    int execute(SymTable symtab) {
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}

class ConstantNode extends Expression {
    int value;

    int execute(SymTable symtab) {
        return value;
    }
}

class VariableNode extends Expression {
    String identifier;

    int execute(SymTable symtab) {
        symtab.lookup(identifier);
    }
}

```

```

program = new Program(
    SubNode(
        AddNode(VariableNode('a'),
            VariableNode('b')),
        ConstantNode(0)
    ),
    SubNode(
        VariableNode('a'),
        VariableNode('c')
    )
)

```

```

void executeProgram(Program program,
    SymTable symtab) {
    int value = program.getExpressions().get(0).execute(s);
    print(value);
    int value = program.getExpressions().get(1).execute(s);
    print(value);
}

```

```

class AddNode extends Expression {
    Expression left;
    Expression right;

    int execute(SymTable symtab) {
        return left.execute() + right.execute();
    }
}

class ConstantNode extends Expression {
    int value;

    int execute(SymTable symtab) {
        return value;
    }
}

class VariableNode extends Expression {
    String identifier;

    int execute(SymTable symtab) {
        symtab.lookup(identifier);
    }
}

```

```

program = new Program(
    SubNode(
        AddNode(VariableNode('a'),
            VariableNode('b')),
        ConstantNode(0)
    ),
    SubNode(
        VariableNode('a'),
        VariableNode('c')
    )
)

```

```

void executeProgram(Program program,
    SymTable symtab) {
    int value =
        program.getExpressions().get(0).left.execute(symtab) -
        program.getExpressions().get(0).right.execute(symtab);
    print(value);
    int value =
        program.getExpressions().get(1).left.execute(symtab) -
        program.getExpressions().get(1).right.execute(symtab);
    print(value);
}

```

```

class AddNode extends Expression {
    Expression left;
    Expression right;

    int execute(SymTable symtab) {
        return left.execute() + right.execute();
    }
}

class ConstantNode extends Expression {
    int value;

    int execute(SymTable symtab) {
        return value;
    }
}

```

```

class VariableNode extends Expression {
    String identifier;

    int execute(SymTable symtab) {
        return symtab.lookup(identifier);
    }
}

```

```

program = new Program(
    SubNode(
        AddNode(VariableNode('a'),
            VariableNode('b')),
        ConstantNode(0)
    ),
    SubNode(
        VariableNode('a'),
        VariableNode('c')
    )
)

```

```

class AddNode extends Expression {
    Expression left;
    Expression right;

    int execute(SymTable symtab) {
        return left.execute() + right.execute();
    }
}

class ConstantNode extends Expression {
    int value;

```

```

void executeProgram(Program program,
    SymTable symtab) {
    int value =
        (program.getExpressions().get(0).left.left.execute(symtab) +
        program.getExpressions().get(0).left.right.execute(symtab)) -
        program.getExpressions().get(0).right.value;
    print(value);
    int value =
        symtable.lookup(program.getExpressions().get(1).left.identifier) -
        symtable.lookup(program.getExpressions().get(1).right.identifier);
    print(value);
}

```

```

(SymTable symtab) {
    value;

    class IdentifierNode extends Expression {
        Identifier identifier;

        int execute(SymTable symtab) {
            return symtable.lookup(identifier);
        }
    }
}

```

```

program = new Program(
    SubNode(
        AddNode(VariableNode('a'),
            VariableNode('b')),
        ConstantNode(0)
    ),
    SubNode(
        VariableNode('a'),
        VariableNode('c')
    )
)

```

```

class AddNode extends Expression {
    Expression left;
    Expression right;

    int execute(SymTable symtab) {
        return left.execute() + right.execute();
    }
}

class ConstantNode extends Expression {
    int value;

```

```

void executeProgram(Program program,
    SymTable symtab) {
    int value =
        (symtable.lookup(program.getExpressions().get(1).left.left.identifier) +
        symtable.lookup(program.getExpressions().get(1).left.right.identifier)) -
        0;
    print(value);
    int value =
        symtable.lookup("a") -
        symtable.lookup("c");
    print(value);
}

```

```

    Table symtab) {

    extends Expression {

    Table symtab) {
        (identifier);

```

```

program = new Program(
    SubNode(
        AddNode(VariableNode('a'),
            VariableNode('b')),
        ConstantNode(0)
    ),
    SubNode(
        VariableNode('a'),
        VariableNode('c')
    )
)

```

```

void executeProgram(Program program,
    SymTable symtab) {
    int value =
        (symtable.lookup("a") +
        symtable.lookup("b")) -
        0;
    print(value);
    int value =
        symtable.lookup("a") -
        symtable.lookup("c");
    print(value);
}

```

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class AddNode extends Expression {
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    SubNode(
        VariableNode('a'),
        VariableNode('c')
    )
)

```

```

void executeProgram(Program program,
    SymTable symtab) {
    int value =
        symtable.lookup("a") +
        symtable.lookup("b");
    print(value);
    int value =
        symtable.lookup("a") -
        symtable.lookup("c");
    print(value);
}

```

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class AddNode extends Expression {
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    int execute(SymTable symtab) {
        return value;
    }
}

class VariableNode extends Expression {
    String identifier;

    int execute(SymTable symtab) {
        return symtab.lookup(identifier);
    }
}

```

Plan

Verification

[specification]

[proof]

Verification

Using
[definitions]

Show
[property]

[proof]

Using
[definitions]

Show

An optimization transformation
preserves the program semantics

[proof]

Using

Program Semantics

Show

An optimization transformation
preserves the program semantics

[proof]

Using

Program Semantics

Optimization Transformation Definition

Show

An optimization transformation
preserves the program semantics

[proof]

Using

Program Semantics

Optimization Transformation Definition

Semantic Preservation Definition

Show

An optimization transformation
preserves the program semantics

[proof]

Using

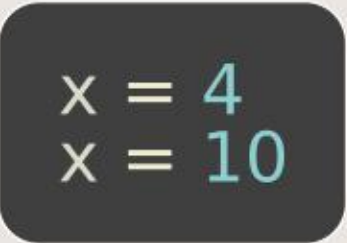
Program Semantics

Optimization Transformation Definition

Semantic Preservation Definition

Show

An optimization transformation
preserves the program semantics



```
x = 4  
x = 10
```

[proof]

Using

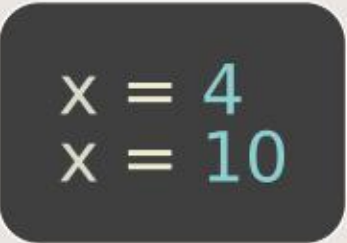
IR Semantics

Optimization Transformation Definition

Semantic Preservation Definition

Show

An optimization transformation
preserves the IR semantics



```
x = 4  
x = 10
```

[proof]

Plan - Implementation

IR Semantics

IR Semantics

Semantics
Preservation
Definition

IR Semantics

Semantics
Preservation
Definition

Isabelle/HOL

IR Semantics

Semantics
Preservation
Definition

Isabelle/HOL



```
graph LR; A[IR Semantics] --> C[Isabelle/HOL]; B[Semantics Preservation Definition] --> C;
```

Optimization Transformation Definitions



Optimization Transformation Definitions



DSL

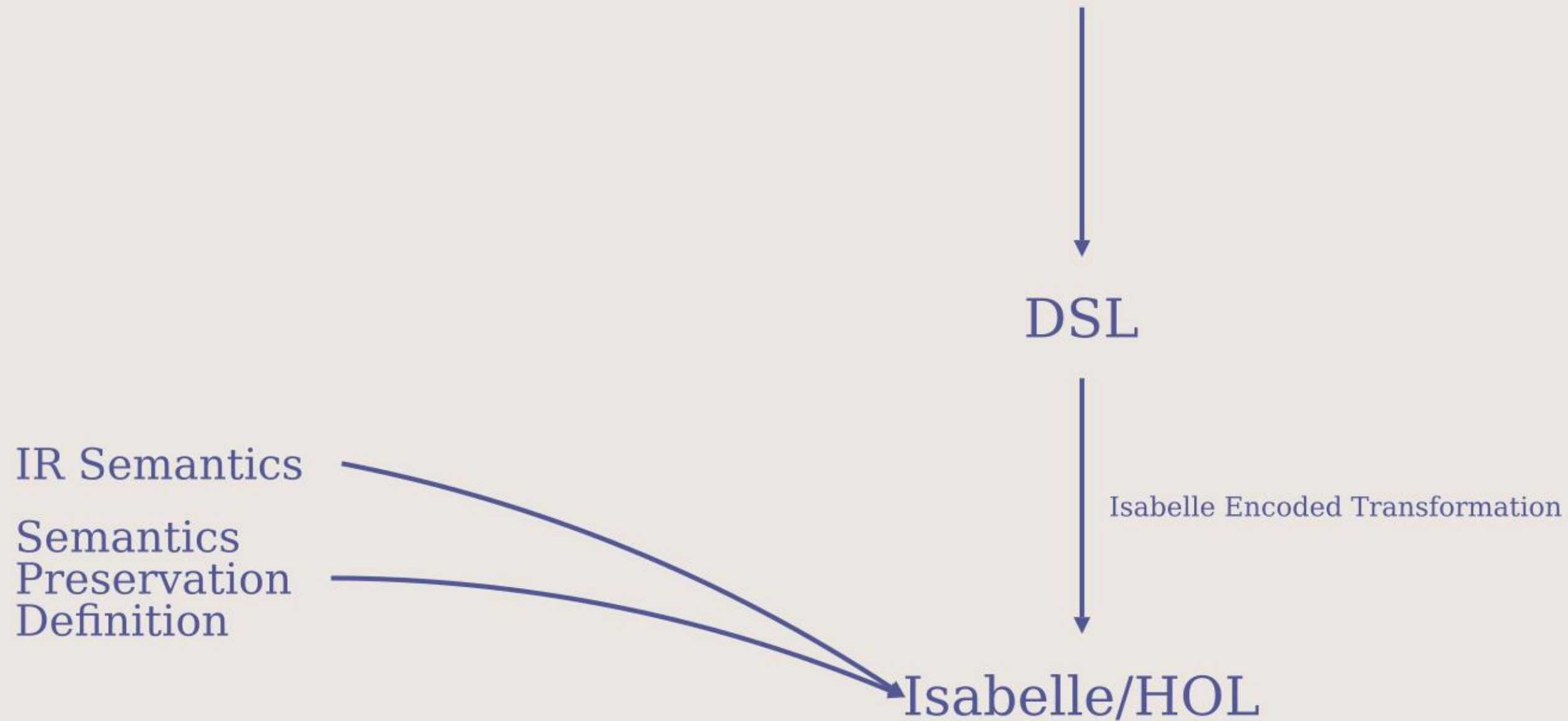
IR Semantics

Semantics
Preservation
Definition

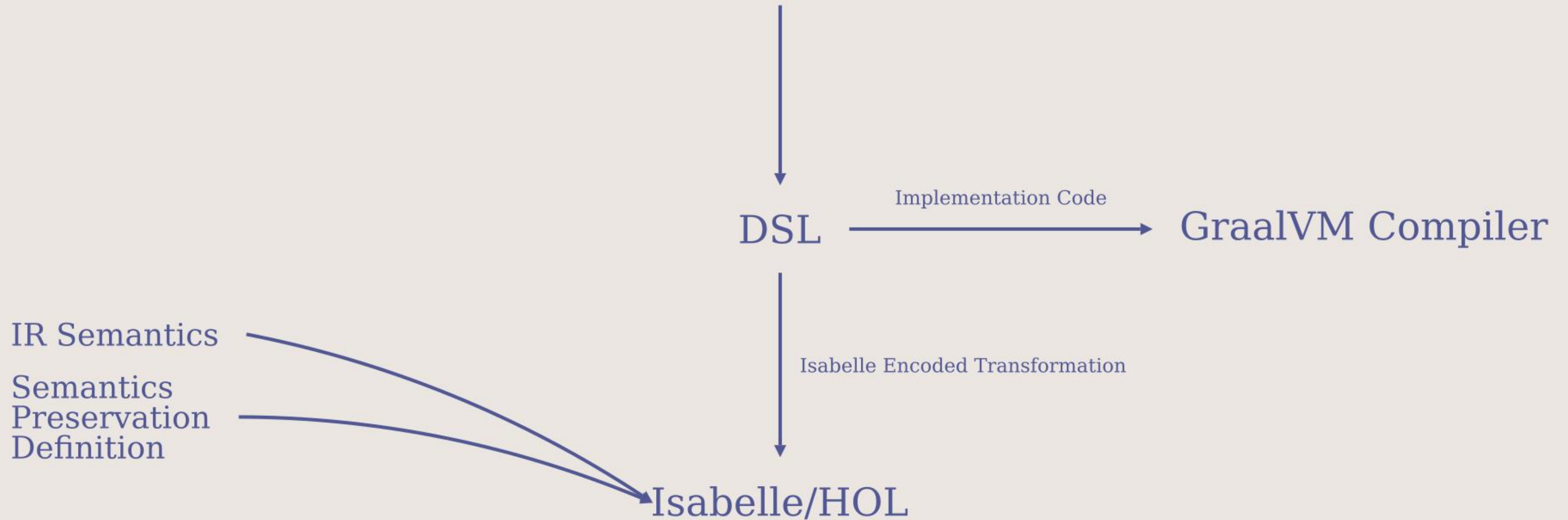
Isabelle/HOL



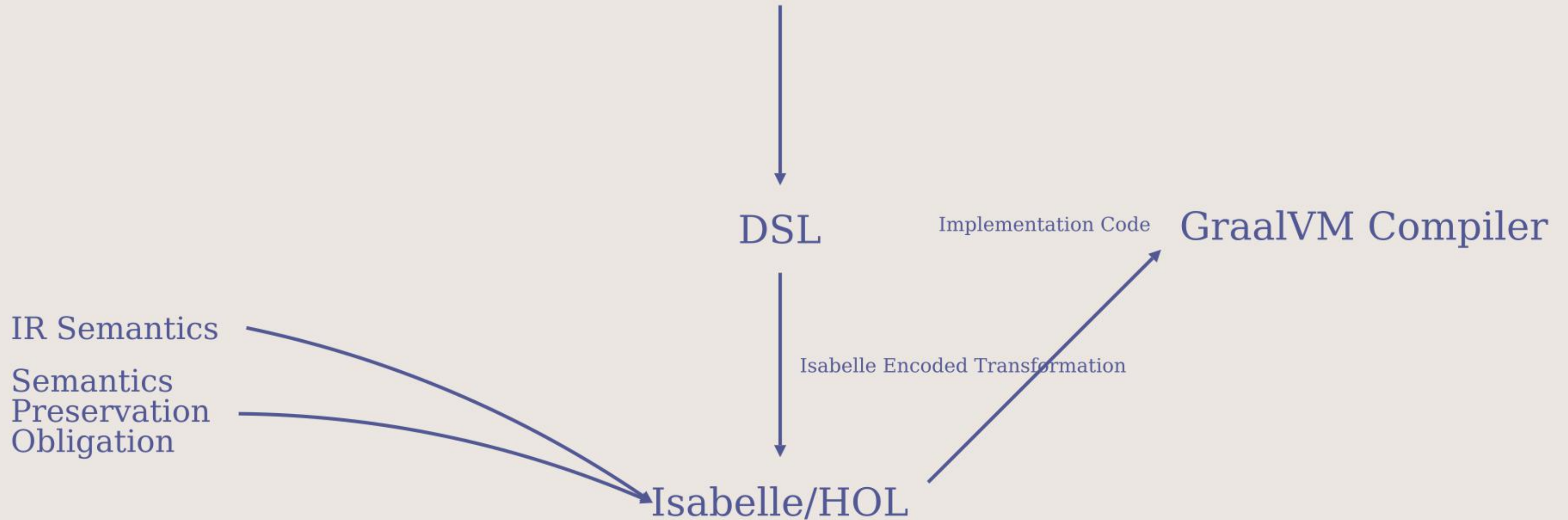
Optimization Transformation Definitions



Optimization Transformation Definitions



Optimization Transformation Definitions



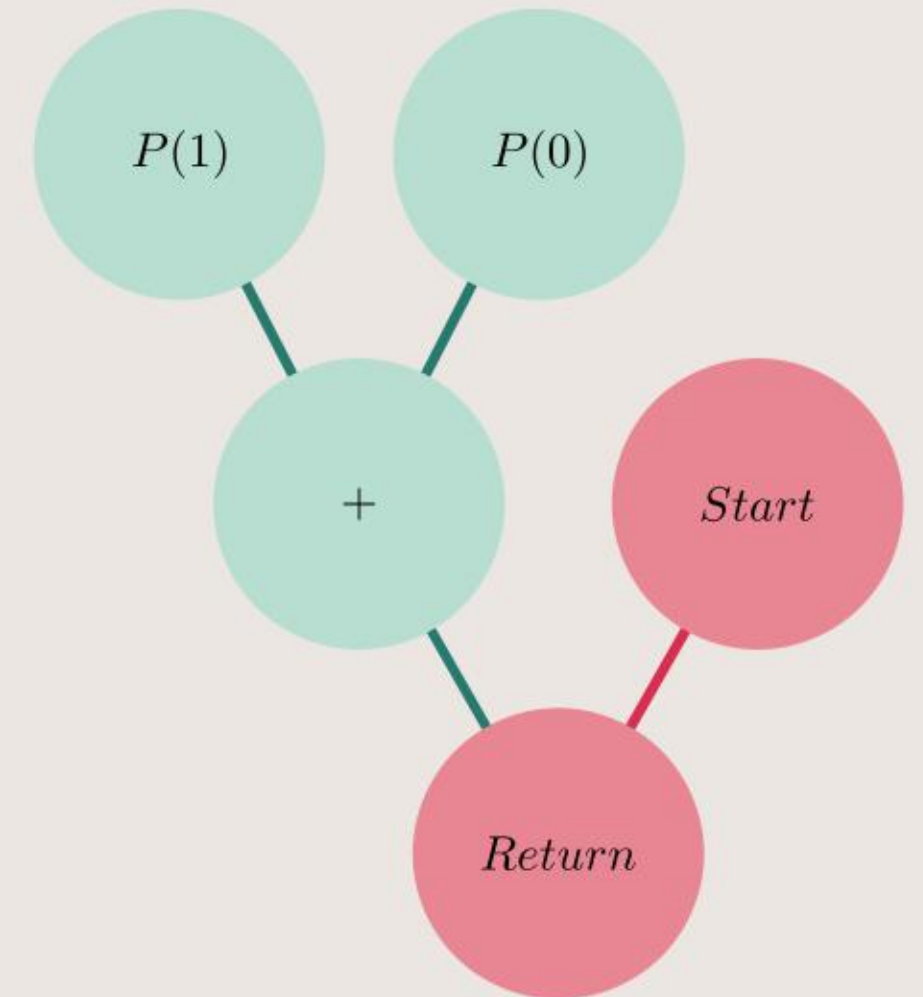
Progress

Cliff Click Michael Paleczny

Workshop on Intermediate Representations
1995

```
public static int add(int x, int y) {  
    return x + y;  
}
```

```
public static int add(int x, int y) {  
    return x + y;  
}
```

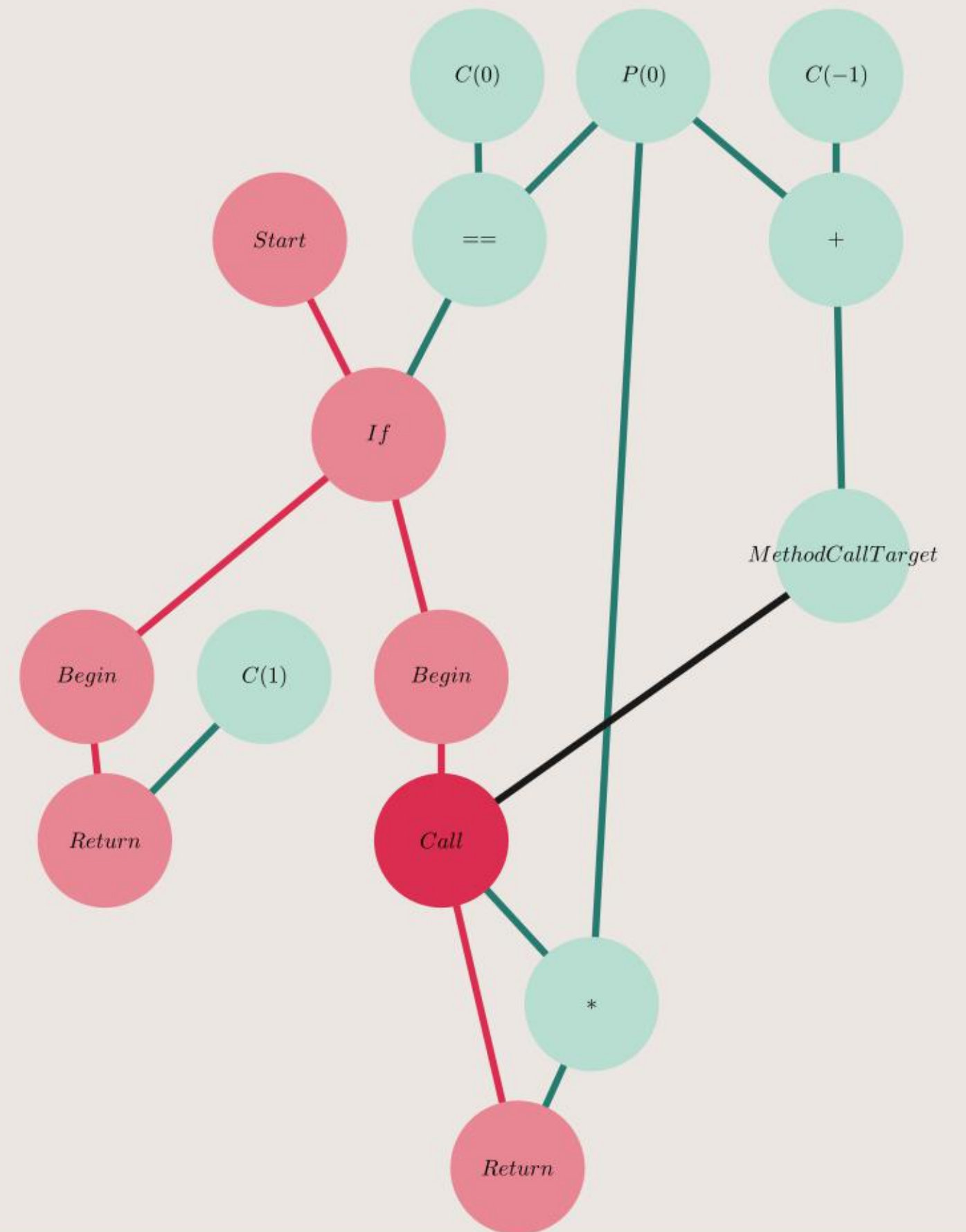


```
int factorial(int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n - 1);  
    }  
}
```

```

int factorial(int n) {
  if (n == 0) {
    return 1;
  } else {
    return n * factorial(n - 1);
  }
}

```



[illegible][illegible][illegible]

$$[g, m, p] \vdash n \mapsto v$$

A Formal Semantics of the GraalVM Intermediate Representation

Brae Webb Mark Utting Ian Hayes

Abstract
The GraalVM intermediate representation (IR) is a
low-level, stack-based, register-based, and
control-flow graph (CFG) based IR.

It is designed to be a common target for
compilers and interpreters, and to be
amenable to optimization and analysis.

This paper presents a formal semantics of the
GraalVM IR, and shows how it can be used to
verify the correctness of optimizations.

The semantics is defined in terms of a
stateful transition system, and is
designed to be amenable to formal
verification.

Symposium On Automated Technology
For Verification And Analysis
2021

$[g, m, p] \vdash n \mapsto v$

$[g, p] \vdash (\text{nid}, m, h) \rightarrow (\text{nid}', m', h')$

A Formal Semantics of the GraalVM Intermediate Representation

Brae Webb Mark Utting Ian Hayes

Abstract

Introduction

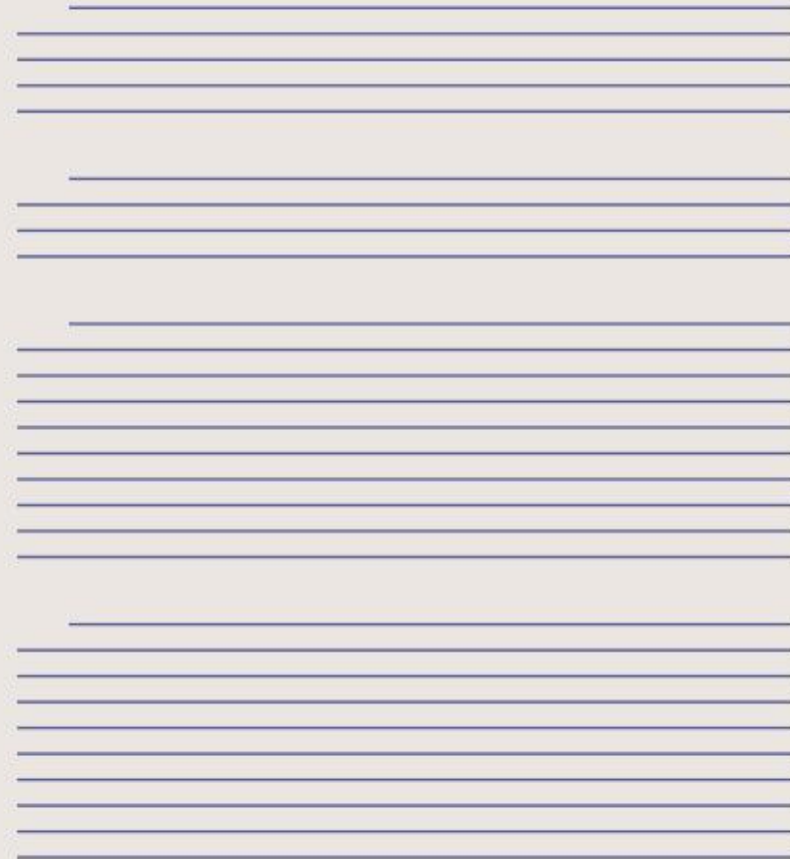
1.1 The GraalVM Intermediate Representation

1.2 The Formal Semantics

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2021

A Formal Semantics of the GraalVM Intermediate Representation

Brae Webb Mark Utting Ian Hayes



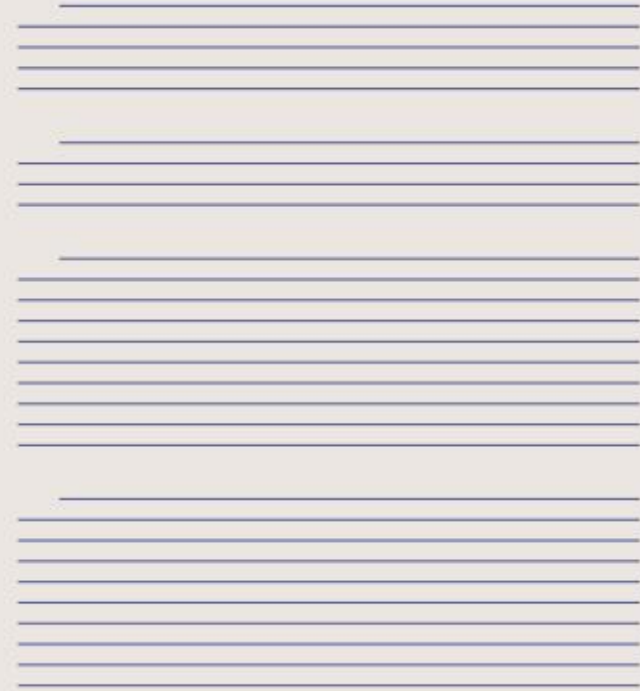
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2021

$$\begin{aligned}
 & [g, m, p] \vdash \text{ConstantNode } c \mapsto c \\
 & \frac{[g, m, p] \vdash g\langle x \rangle \mapsto v1 \quad [g, m, p] \vdash g\langle y \rangle \mapsto v2}{[g, m, p] \vdash \text{AddNode } x \ y \mapsto v1 + v2} \\
 & [g, m, p] \vdash \text{ParameterNode } i \mapsto p[i] \\
 & [g, m, p] \vdash \text{ValuePhiNode } nid \ uu \ uv \mapsto m \ nid \\
 & [g, m, p] \vdash \text{InvokeNode } nid \ vh \ vi \ vj \ vk \ vl \mapsto m \ nid \\
 & [g, m, p] \vdash \text{NewInstanceNode } nid \ vs \ vt \ vu \mapsto m \ nid \\
 & [g, m, p] \vdash \text{LoadFieldNode } nid \ vv \ vw \ vx \mapsto m \ nid
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\text{is-sequential-node } g\langle nid \rangle \quad nid' = (\text{successors-of } g\langle nid \rangle)_{[0]}}{[g, p] \vdash (nid, m, h) \rightarrow (nid', m, h)} \\
 & \frac{g\langle nid \rangle = \text{IfNode } cond \ tb \ fb \quad [g, m, p] \vdash g\langle cond \rangle \mapsto val \quad nid' = (\text{if } val\text{-to-bool } val \text{ then } tb \text{ else } fb)}{[g, p] \vdash (nid, m, h) \rightarrow (nid', m, h)} \\
 & \frac{\text{is-AbstractEndNode } g\langle nid \rangle \quad \text{merge} = \text{any-usage } g \ nid \quad \text{is-AbstractMergeNode } g\langle \text{merge} \rangle \quad i = \text{find-index } nid \ (\text{inputs-of } g\langle \text{merge} \rangle) \quad phis = \text{phi-list } g \ \text{merge} \quad inps = \text{phi-inputs } g \ i \ phis \quad [g, m, p] \vdash inps \mapsto vs \quad m' = \text{set-phis } phis \ vs \ m}{[g, p] \vdash (nid, m, h) \rightarrow (\text{merge}, m', h)} \\
 & \frac{g\langle nid \rangle = \text{NewInstanceNode } nid \ class \ frame \ nid' \quad (h', ref) = h\text{-new-inst } h \quad m' = m(nid := ref)}{[g, p] \vdash (nid, m, h) \rightarrow (nid', m', h')} \\
 & \frac{g\langle nid \rangle = \text{LoadFieldNode } nid \ f \ (\text{Some } obj) \ nid' \quad [g, m, p] \vdash g\langle obj \rangle \mapsto \text{ObjRef } ref \quad h\text{-load-field } ref \ f \ h = v \quad m' = m(nid := v)}{[g, p] \vdash (nid, m, h) \rightarrow (nid', m', h)} \\
 & \frac{g\langle nid \rangle = \text{StoreFieldNode } nid \ f \ newval \ uu \ (\text{Some } obj) \ nid' \quad [g, m, p] \vdash g\langle newval \rangle \mapsto val \quad [g, m, p] \vdash g\langle obj \rangle \mapsto \text{ObjRef } ref \quad h' = h\text{-store-field } ref \ f \ val \ h \quad m' = m(nid := val)}{[g, p] \vdash (nid, m, h) \rightarrow (nid', m', h')}
 \end{aligned}$$

A Formal Semantics of the GraalVM Intermediate Representation

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Symposium On Automated Technology
For Verification And Analysis
2021

$$\begin{aligned}
 & [g, m, p] \vdash \text{ConstantNode } c \mapsto c \\
 & \frac{[g, m, p] \vdash g\langle x \rangle \mapsto v1 \quad [g, m, p] \vdash g\langle y \rangle \mapsto v2}{[g, m, p] \vdash \text{AddNode } x \ y \mapsto v1 + v2} \\
 & [g, m, p] \vdash \text{ParameterNode } i \mapsto p[i] \\
 & [g, m, p] \vdash \text{ValuePhiNode } nid \ uu \ uv \mapsto m \ nid \\
 & [g, m, p] \vdash \text{InvokeNode } nid \ vh \ vi \ vj \ vk \ vl \mapsto m \ nid \\
 & [g, m, p] \vdash \text{NewInstanceNode } nid \ vs \ vt \ vu \mapsto m \ nid \\
 & [g, m, p] \vdash \text{LoadFieldNode } nid \ vv \ vw \ vx \mapsto m \ nid
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\text{is-sequential-node } g\langle nid \rangle \quad nid' = (\text{successors-of } g\langle nid \rangle)_{[0]}}{[g, p] \vdash (nid, m, h) \rightarrow (nid', m, h)} \\
 & \frac{g\langle nid \rangle = \text{IfNode } cond \ tb \ fb \quad [g, m, p] \vdash g\langle cond \rangle \mapsto val \quad nid' = (\text{if val-to-bool val then tb else fb})}{[g, p] \vdash (nid, m, h) \rightarrow (nid', m, h)} \\
 & \frac{\text{is-AbstractEndNode } g\langle nid \rangle \quad \text{merge} = \text{any-usage } g \ nid \quad \text{is-AbstractMergeNode } g\langle \text{merge} \rangle \quad i = \text{find-index } nid \ (\text{inputs-of } g\langle \text{merge} \rangle) \quad phis = \text{phi-list } g \ \text{merge} \quad inps = \text{phi-inputs } g \ i \ phis \quad [g, m, p] \vdash inps \mapsto vs \quad m' = \text{set-phis } phis \ vs \ m}{[g, p] \vdash (nid, m, h) \rightarrow (\text{merge}, m', h)} \\
 & \frac{g\langle nid \rangle = \text{NewInstanceNode } nid \ class \ frame \ nid' \quad (h', ref) = h\text{-new-inst } h \quad m' = m(nid := ref)}{[g, p] \vdash (nid, m, h) \rightarrow (nid', m', h')} \\
 & \frac{g\langle nid \rangle = \text{LoadFieldNode } nid \ f \ (\text{Some } obj) \ nid' \quad [g, m, p] \vdash g\langle obj \rangle \mapsto \text{ObjRef } ref \quad h\text{-load-field } ref \ f \ h = v \quad m' = m(nid := v)}{[g, p] \vdash (nid, m, h) \rightarrow (nid', m', h)} \\
 & \frac{g\langle nid \rangle = \text{StoreFieldNode } nid \ f \ newval \ uu \ (\text{Some } obj) \ nid' \quad [g, m, p] \vdash g\langle newval \rangle \mapsto val \quad [g, m, p] \vdash g\langle obj \rangle \mapsto \text{ObjRef } ref \quad h' = h\text{-store-field } ref \ f \ val \ h \quad m' = m(nid := val)}{[g, p] \vdash (nid, m, h) \rightarrow (nid', m', h')}
 \end{aligned}$$

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$$x + y - y \rightarrow x$$

$$x + y - y \rightarrow x$$

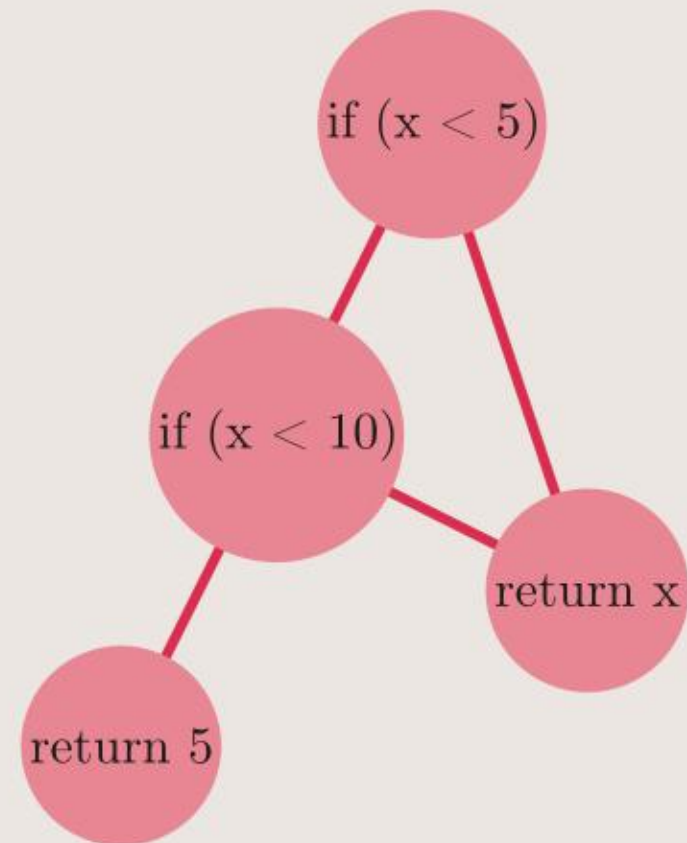
$$\forall s. (\text{eval } (node_1, s) = v) \leftrightarrow (\text{eval } (node_2, s) = v)$$

```
if (x < 5) {  
    if (x < 10) {  
        return 5;  
    }  
}  
return x;
```

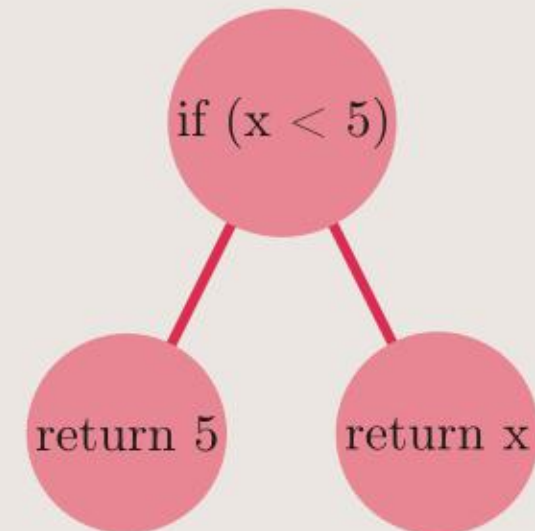
```
if (x < 5) {  
    if (x < 10) {  
        return 5;  
    }  
}  
return x;
```

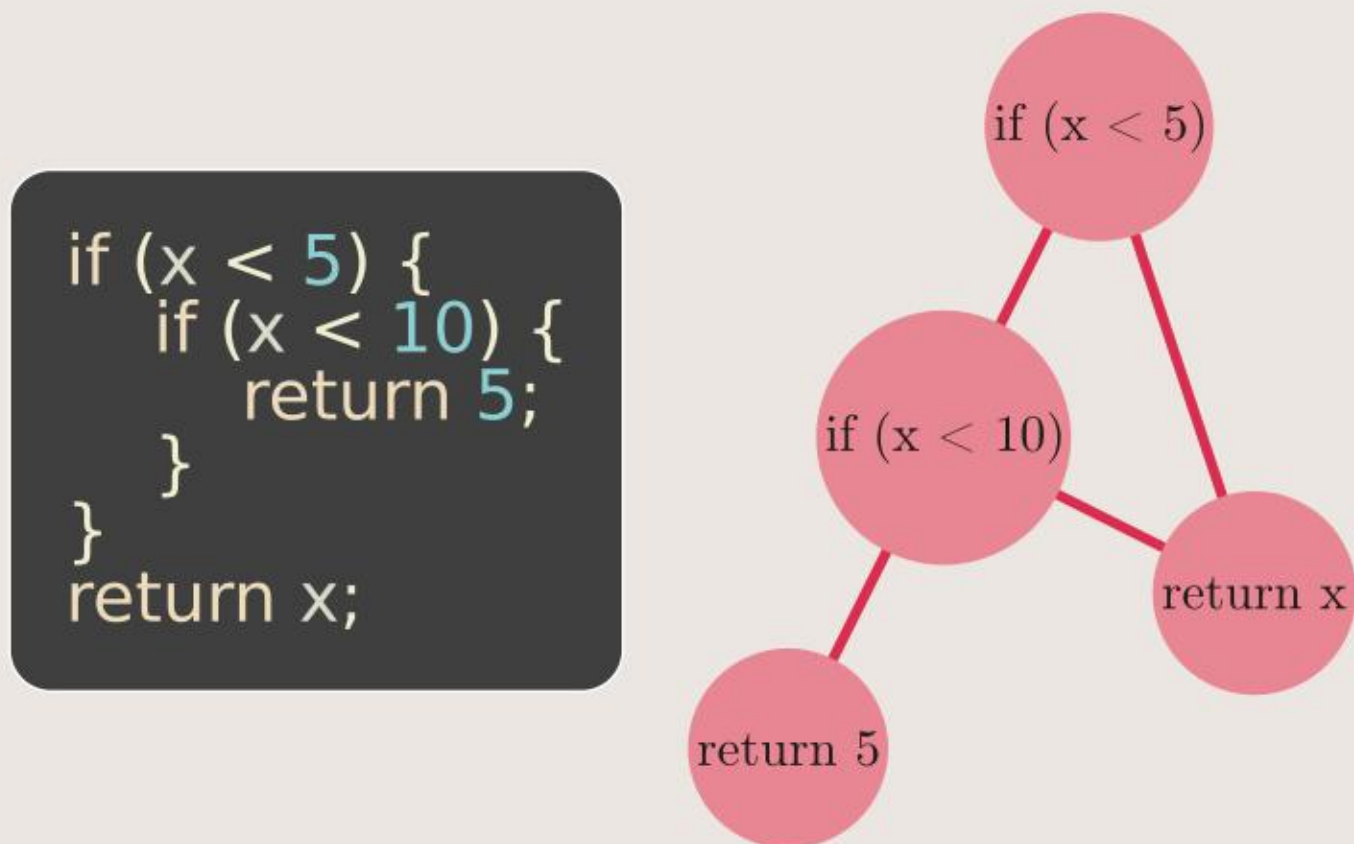
```
if (x < 5) {  
    return 5;  
}  
return x;
```

```
if (x < 5) {  
  if (x < 10) {  
    return 5;  
  }  
}  
return x;
```



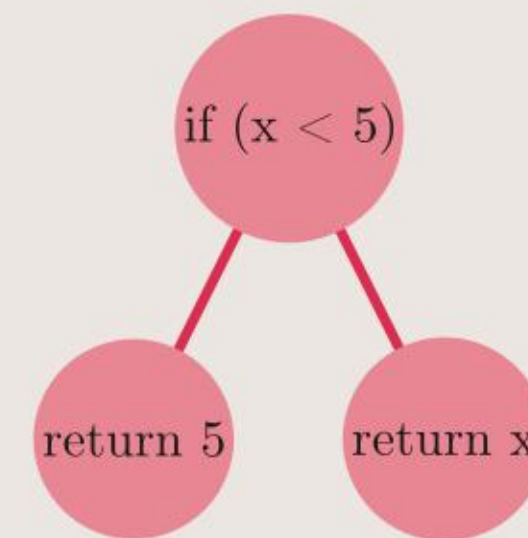
```
if (x < 5) {  
  return 5;  
}  
return x;
```



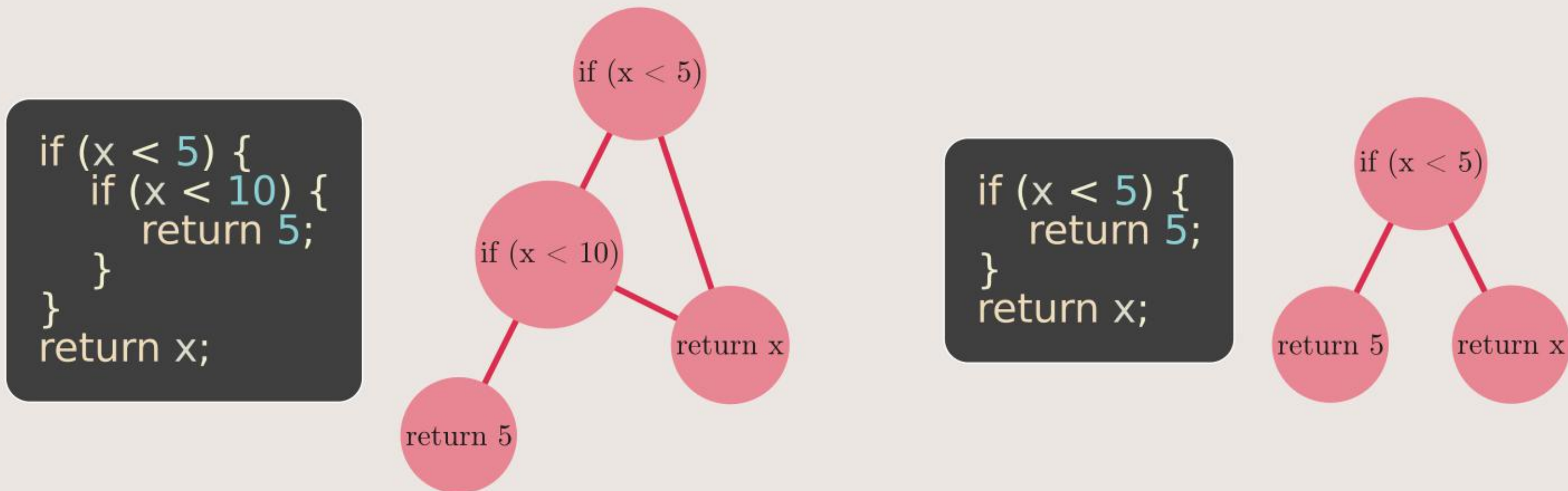


```

if (x < 5) {
  return 5;
}
return x;
  
```

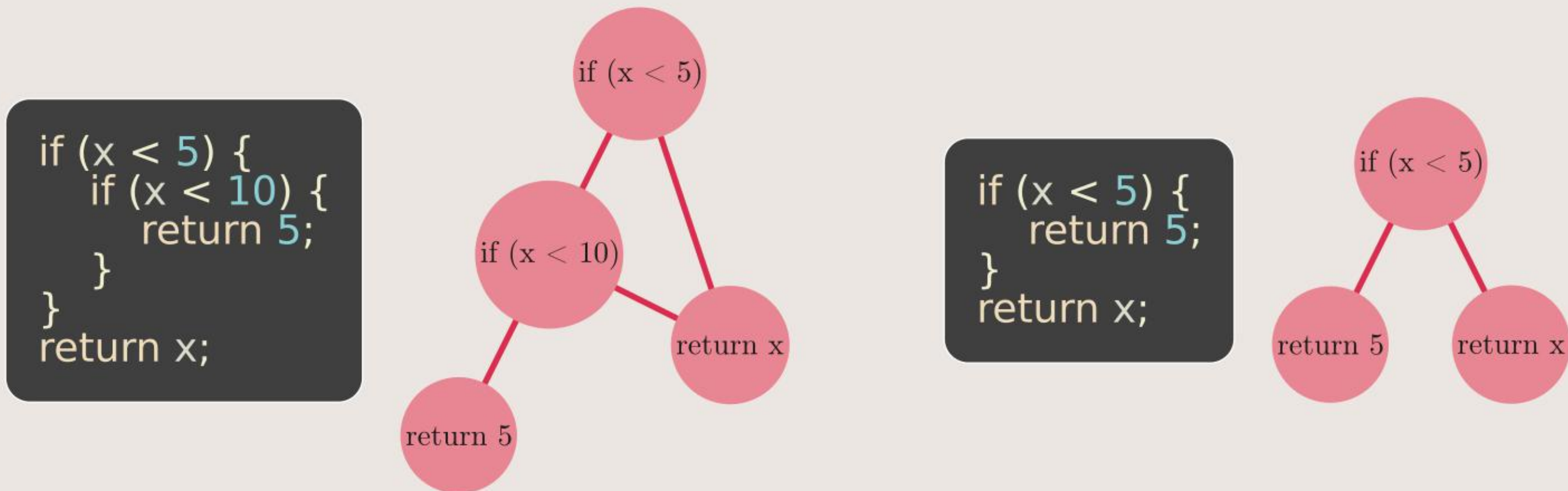


$$P \sim Q \Leftrightarrow ((\forall P'. P \rightsquigarrow P' \rightarrow (\exists Q'. Q \rightsquigarrow Q' \wedge P' \sim Q')) \wedge (\forall Q'. Q \rightsquigarrow Q' \rightarrow (\exists P'. P \rightsquigarrow P' \wedge P' \sim Q)))$$



$$P \sim Q \Leftrightarrow ((\forall P'. P \rightsquigarrow P' \rightarrow (\exists Q'. Q \rightsquigarrow Q' \wedge P' \sim Q')) \wedge (\forall Q'. Q \rightsquigarrow Q' \rightarrow (\exists P'. P \rightsquigarrow P' \wedge P' \sim Q')))$$

$$\frac{\forall P'. [g, p] \vdash (nid, m, h) \rightarrow P' \longrightarrow (\exists Q'. [g', p] \vdash (nid, m, h) \rightarrow Q' \wedge P' = Q') \quad \forall Q'. [g', p] \vdash (nid, m, h) \rightarrow Q' \longrightarrow (\exists P'. [g, p] \vdash (nid, m, h) \rightarrow P' \wedge P' = Q')}{nid \ m \ h \mid g \sim g'}$$



$$P \sim Q \Leftrightarrow ((\forall P'. P \rightsquigarrow P' \rightarrow (\exists Q'. Q \rightsquigarrow Q' \wedge P' \sim Q')) \wedge (\forall Q'. Q \rightsquigarrow Q' \rightarrow (\exists P'. P \rightsquigarrow P' \wedge P' \sim Q)))$$

$$\frac{\forall P'. [g, p] \vdash (nid, m, h) \rightarrow P' \longrightarrow (\exists Q'. [g', p] \vdash (nid, m, h) \rightarrow Q' \wedge P' = Q') \quad \forall Q'. [g', p] \vdash (nid, m, h) \rightarrow Q' \longrightarrow (\exists P'. [g, p] \vdash (nid, m, h) \rightarrow P' \wedge P' = Q')}{nid \ m \ h \mid g \sim g'}$$

$$\frac{\forall P'. [g, m, p, h] \vdash nid \rightsquigarrow P' \longrightarrow (\exists Q'. [g', m, p, h] \vdash nid \rightsquigarrow Q' \wedge P' = Q') \quad \forall Q'. [g', m, p, h] \vdash nid \rightsquigarrow Q' \longrightarrow (\exists P'. [g, m, p, h] \vdash nid \rightsquigarrow P' \wedge P' = Q')}{nid . g \sim g'}$$

Canonicalization Phase

Canonicalization Phase

$stampx = stamp\text{-}expr\ x$	$stampx = IntegerStamp\ 32\ lo\ hi$
$CanonicalizeSub\ (BinaryExpr\ BinSub\ x\ x)\ (ConstantExpr\ (IntVal32\ 0))$	
$stampx = stamp\text{-}expr\ x$	$stampx = IntegerStamp\ 64\ lo\ hi$
$CanonicalizeSub\ (BinaryExpr\ BinSub\ x\ x)\ (ConstantExpr\ (IntVal64\ 0))$	
$x = BinaryExpr\ BinAdd\ a\ b$	
$stampa = stamp\text{-}expr\ a$	$stampb = stamp\text{-}expr\ b$
$is\ IntegerStamp\ stampa \wedge is\ IntegerStamp\ stampb$	
$stampc = IntegerStamp\ 32\ stp\ bits$	$stampx = stamp\text{-}expr\ x$
$CanonicalizeSub\ (BinaryExpr\ BinSub\ x\ x)\ (ConstantExpr\ (IntVal32\ 0))$	
$is\ IntegerStamp\ stampa \wedge is\ IntegerStamp\ stampy$	
$x = BinaryExpr\ BinAdd\ a\ b$	
$stampa = stamp\text{-}expr\ a$	$stampb = stamp\text{-}expr\ b$
$c = ConstantExpr\ val$	$IRTreeEval.val\ to\ bool\ val$
$CanonicalizeConditional\ (ConditionalExpr\ c\ t\ f)\ t$	
$CanonicalizeSub\ (BinaryExpr\ BinSub\ x\ a)\ b$	
$c = ConstantExpr\ val$	$\neg IRTreeEval.val\ to\ bool\ val$
$x = BinaryExpr\ BinSub\ a\ b$	
$stampa = stamp\text{-}expr\ a$	$stampb = stamp\text{-}expr\ b$
$is\ IntegerStamp\ stampa \wedge is\ IntegerStamp\ stampb$	
$CanonicalizeIntegerEquals\ (BinaryExpr\ BinIntegerEquals\ x\ y)\ (ConstantExpr\ (IntVal32\ 1))$	
$CanonicalizeSub\ (BinaryExpr\ BinSub\ x\ a)\ (UnaryExpr\ UnaryNeg\ b)$	
$alwaysDistinct\ (stamp\text{-}expr\ x)\ (stamp\text{-}expr\ y)$	
$x = BinaryExpr\ BinAdd\ a\ b$	
$stampa = stamp\text{-}expr\ a$	$stampb = stamp\text{-}expr\ b$
$is\ IntegerStamp\ stampa \wedge is\ IntegerStamp\ stampb$	
$stp\ bits\ stampa = stp\ bits\ stampb$	
$CanonicalizeSub\ (BinaryExpr\ BinSub\ a\ b)\ (UnaryExpr\ UnaryNeg\ x)$	
$y = BinaryExpr\ BinAdd\ x\ y$	
$stampa = stamp\text{-}expr\ a$	$stampb = stamp\text{-}expr\ b$
$is\ IntegerStamp\ stampa \wedge is\ IntegerStamp\ stampb$	
$stp\ bits\ stampa = stp\ bits\ stampb$	
$CanonicalizeIntegerEquals\ (BinaryExpr\ BinIntegerEquals\ left\ right)\ (BinaryExpr\ BinIntegerEquals\ y\ z)$	
$stp\ bits\ stampa = stp\ bits\ stampb$	
$x = BinaryExpr\ BinSub\ a\ b$	
$stampa = stamp\text{-}expr\ a$	$stampb = stamp\text{-}expr\ b$
$is\ IntegerStamp\ stampa \wedge is\ IntegerStamp\ stampb$	
$stp\ bits\ stampa = stp\ bits\ stampb$	
$CanonicalizeSub\ (BinaryExpr\ BinSub\ a\ y)\ b$	
$stampx = stamp\text{-}expr\ x$	
$stampx = IntegerStamp\ 32\ lo\ hi$	
$CanonicalizeSub\ (BinaryExpr\ BinSub\ (ConstantExpr\ (IntVal32\ 0))\ x)\ (UnaryExpr\ UnaryNeg\ x)$	

[illegible]

	$stampx = stamp\text{-}expr\ x$	$stampx = IntegerStamp\ 64\ lo\ hi$
CanonicalizeSub	$(BinaryExpr\ BinSub\ (ConstantExpr\ (IntVal64\ 0))\ x)\ (UnaryExpr\ UnaryNeg\ x)$	
	$nb = UnaryExpr\ BinaryExpr\ BinAdd\ y\ x$	
	$stampa = stamp\text{-}expr\ x$	$stampr = BinaryExpr\ BinAdd\ expr\ b$
CanonicalizeIntegerEquals	$(BinaryExpr\ BinIntegerEquals\ left\ right)\ (BinaryExpr\ BinIntegerEquals\ y\ z)$	
	$stp\text{-}bits\ stampa = stp\text{-}bits\ stampr$	
CanonicalizeSub	$(BinaryExpr\ BinSub\ a\ b)\ (BinaryExpr\ BinAdd\ a\ b)$	
	$right = BinaryExpr\ BinSub\ x\ z$	
CanonicalizeIntegerEquals	$(BinaryExpr\ BinIntegerEquals\ left\ right)\ (BinaryExpr\ BinIntegerEquals\ y\ z)$	
	$stamp\text{-}expr\ x = IntegerStamp\ 32\ lo\ hi$	
CanonicalizeMul	$(BinaryExpr\ BinMul\ x\ y)\ (UnaryExpr\ UnaryNeg\ x)$	
	$right = BinaryExpr\ BinSub\ z\ x$	
CanonicalizeIntegerEquals	$(BinaryExpr\ BinIntegerEquals\ left\ right)\ (BinaryExpr\ BinIntegerEquals\ y\ z)$	
	$stamp\text{-}expr\ x = IntegerStamp\ 64\ lo\ hi$	
CanonicalizeMul	$(BinaryExpr\ BinMul\ x\ y)\ (UnaryExpr\ UnaryNeg\ x)$	
	$zeroa = ConstantExpr\ (IntVal32\ 0)$	
CanonicalizeIntegerEquals	$(BinaryExpr\ BinIntegerEquals\ left\ x)\ (BinaryExpr\ BinIntegerEquals\ y\ zeroa)$	
y))	CanonicalizeUnaryOp	$(UnaryExpr\ op\ (ConstantExpr\ val))\ (ConstantExpr\ val)$
	$left = BinaryExpr\ BinAdd\ x\ y$	
	$x = ConstantExpr\ zeroa$	$zeroa = ConstantExpr\ (IntVal32\ 0)$
CanonicalizeIntegerEquals	$(BinaryExpr\ BinIntegerEquals\ left\ x)\ (BinaryExpr\ BinIntegerEquals\ x\ zeroa)$	
	CanonicalizeBinaryOp	$(BinaryExpr\ op\ x\ y)\ (ConstantExpr\ val)$
	$right = BinaryExpr\ BinAdd\ x\ y$	
	$y = ConstantExpr\ zeroa$	$zeroa = ConstantExpr\ (IntVal32\ 0)$
CanonicalizeIntegerEquals	$(BinaryExpr\ BinIntegerEquals\ left\ right)\ (BinaryExpr\ BinIntegerEquals\ y\ zeroa)$	
	$stampy = stamp\text{-}expr\ y$	$stp\text{-}bits\ stampa = stp\text{-}bits\ stampy$
	$is\text{-}IntegerStamp\ stampy$	$is\text{-}IntegerStamp\ stampy$
	CanonicalizeBinaryOp	$(BinaryExpr\ op\ x\ y)\ (ConstantExpr\ (IntVal32\ 0))$
	$zeroa = ConstantExpr\ (IntVal32\ 0)$	
CanonicalizeIntegerEquals	$(BinaryExpr\ BinIntegerEquals\ y\ right)\ (BinaryExpr\ BinIntegerEquals\ x\ zeroa)$	
	$y = ConstantExpr\ c$	$is\text{-}zero\ op\ c$
	$stampx = stamp\text{-}expr\ x$	$stampr = BinaryExpr\ BinSub\ expr\ y$
	$stp\text{-}bits\ stampx = stp\text{-}bits\ stampy$	$stp\text{-}bits\ stampx = stp\text{-}bits\ stampy$
CanonicalizeIntegerEquals	$(BinaryExpr\ BinIntegerEquals\ left\ right)\ (BinaryExpr\ BinIntegerEquals\ y\ zeroa)$	
	CanonicalizeBinaryOp	$(BinaryExpr\ op\ x\ y)\ (ConstantExpr\ (IntVal32\ 0))$
	$right = BinaryExpr\ BinSub\ x\ y$	
	$y = ConstantExpr\ zeroa$	$zeroa = ConstantExpr\ (IntVal32\ 0)$
CanonicalizeIntegerEquals	$(BinaryExpr\ BinIntegerEquals\ left\ right)\ (BinaryExpr\ BinIntegerEquals\ y\ zeroa)$	
	$stampx = stamp\text{-}expr\ x$	$stampy = stamp\text{-}expr\ y$
	$stp\text{-}bits\ stampx = stp\text{-}bits\ stampy$	$stp\text{-}bits\ stampx = 64$
	$is\text{-}IntegerStamp\ stampx \wedge is\text{-}IntegerStamp\ stampy$	
CanonicalizeBinaryOp	$(BinaryExpr\ op\ x\ y)\ (ConstantExpr\ (IntVal64\ 0))$	

Conditional Elimination Phase

Conditional Elimination Phase

$\text{alwaysDistinct (stamps } x) \text{ (stamps } y)$	$g \vdash \text{IntegerEqualsNode } x \ y \ \& \ \text{IntegerLessThanNode } x \ y \mapsto \text{KnownFalse}$
$\text{tryFold (IntegerEqualsNode } x \ y) \text{ stamps KnownFalse}$	$g \vdash \text{IntegerEqualsNode } x \ y \ \& \ \text{IntegerLessThanNode } y \ x \mapsto \text{KnownFalse}$
$\text{neverDistinct (stamps } x) \text{ (stamps } y)$	$g \vdash \text{IntegerLessThanNode } x \ y \ \& \ \text{IntegerLessThanNode } y \ x \mapsto$
$\text{tryFold (IntegerEqualsNode } x \ y) \text{ stamps KnownTrue}$	$g \llbracket \text{nid} \rrbracket = \text{BeginNode nid}' \quad \text{KnownFalse} \quad \text{nid} \notin \text{seen} \quad \text{seen}' = \{\text{nid}\} \cup \text{seen}$
$\text{is-IntegerStamp (stamps } x) \quad \text{is-IntegerStamp (stamps } y)$	$g \vdash \text{IntegerLessThanNode } x \ y \ \& \ \text{IntegerEqualsNode pred } g \ \text{nid} \mapsto \text{KnownFalse}$
$\text{stpi-upper (stamps } x) < \text{stpi-lower (stamps } y)$	$g \llbracket \text{ifcond} \rrbracket = \text{IfNode cond } t \ f$
$\text{tryFold (IntegerLessThanNode } x \ y) \text{ stamps KnownTrue}$	$g \vdash \text{IntegerLessThanNode } x \ y \ \& \ \text{IntegerEqualsNode } y \ x \mapsto \text{KnownFalse}$
$\text{is-IntegerStamp (stamps } x) \quad \text{is-IntegerStamp (stamps } y)$	$c = (\text{if } i = 0 \text{ then } g \llbracket \text{cond} \rrbracket \text{ else } \text{LogicNegationNode cond})$
$\text{stpi-upper (stamps } y) \leq \text{stpi-lower (stamps } x)$	$g \vdash x \ \& \ x \mapsto \text{KnownTrue}$
$\text{ConditionalElimination.Step } g \text{ (nid, seen, conds, flow) (Some (nid', seen', conds', flow'))}$	$\text{conds}' = c \cdot \text{conds}$
$\text{ConditionalElimination.Step (set conds) (hdOr flow (stamp } g)) \ g \ \text{nid } g'$	$\text{flow}' = \text{registerNewCondition } g \ \& \ g \llbracket y \rrbracket \text{ (hdOr flow (stamp } g))$
$\text{ConditionalEliminationPhase } g' \text{ (nid', seen', conds', flow')}$	$g \vdash x \ \& \ g \llbracket y \rrbracket \mapsto \text{KnownTrue}$
$\text{ConditionalEliminationPhase } g \text{ (nid, seen, conds, flow) } g' \text{ nid' = any-usage } g \ \text{nid}$	$g \vdash x \ \& \ \text{LogicNegationNode } y \mapsto \text{KnownFalse}$
$\text{ConditionalEliminationPhase } g \text{ (nid, seen, conds, flow) (Some (nid', seen', conds', flow'))}$	$g \vdash x \ \& \ \text{LogicNegationNode } y \mapsto \text{KnownTrue}$
$\text{ConditionalEliminationPhase } g \text{ (nid', seen', conds', flow') } g'$	$\neg \text{is-EndNode } g \llbracket \text{nid} \rrbracket$
$\text{ConditionalEliminationPhase } g \text{ (nid, seen, conds, flow) } g' \text{ is-BeginNode } g \llbracket \text{nid} \rrbracket$	$\text{nid} \notin \text{seen} \quad \text{seen}' = \{\text{nid}\} \cup \text{seen}$
$\text{ConditionalEliminationPhase } g \text{ (nid, seen, conds, flow) } g' \text{ None = ConditionalElimination.nextEdge seen' nid } g$	$\text{Some nid}' = \text{ConditionalElimination.nextEdge seen' nid } g$
$\text{ConditionalEliminationPhase } g \text{ (nid, seen, conds, flow) } g' \text{ None = ConditionalElimination.pred } g \ \text{nid}$	$\neg \text{is-EndNode } g \llbracket \text{nid} \rrbracket$
$\text{ConditionalEliminationPhase } g \text{ (nid, seen, conds, flow) } g' \text{ None = ConditionalElimination.pred } g \ \text{nid}$	$\text{nid} \in \text{seen}$
$\text{ConditionalEliminationPhase } g \text{ (nid, seen, conds, flow) } g \text{ ConditionalElimination.Step } g \text{ (nid, seen, conds, flow) None}$	

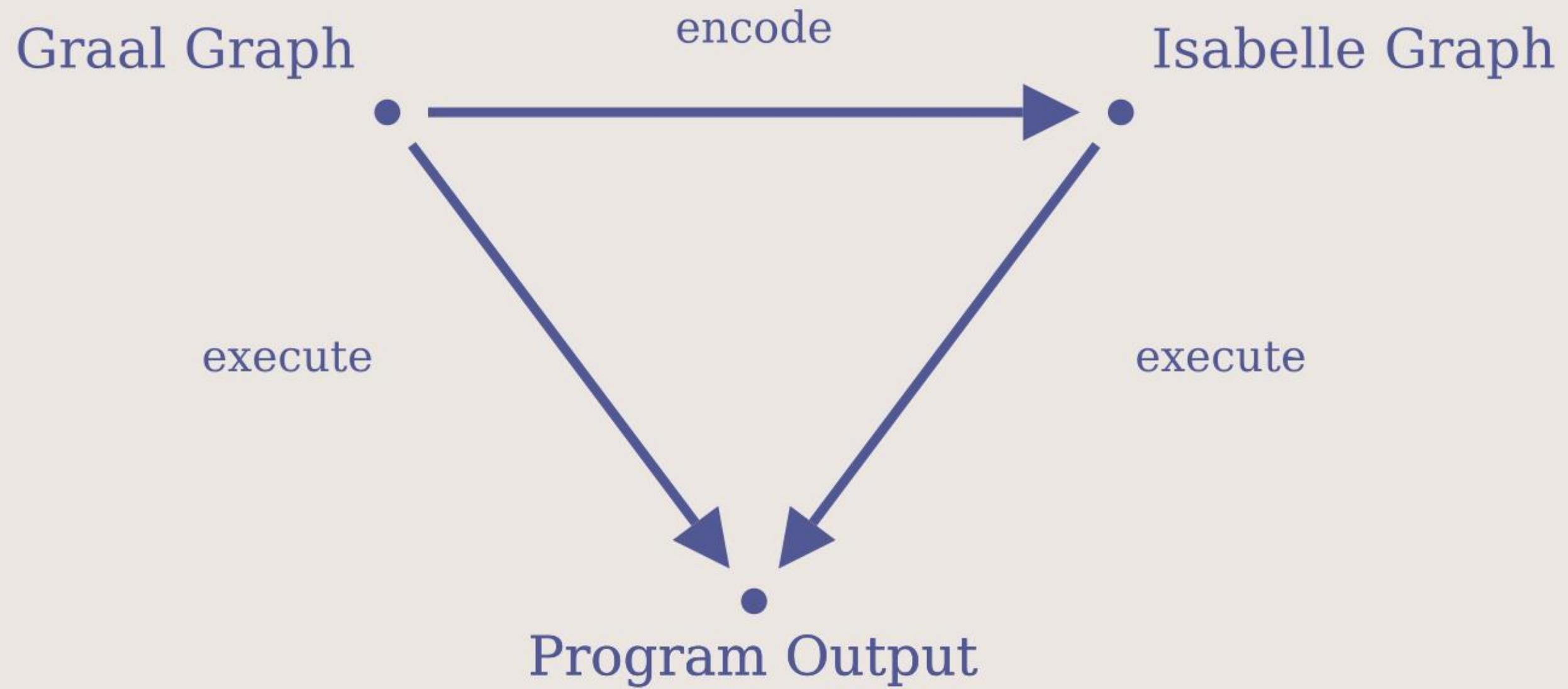
$$\begin{array}{c}
g\langle\langle\text{ifcond}\rangle\rangle = \text{IfNode } cid \ t \ f \\
\text{cond} = g\langle\langle cid \rangle\rangle \quad \exists c \in \text{conds. } g \vdash c \ \& \ \text{cond} \mapsto \text{KnownTrue} \\
g' = \text{constantCondition } \text{True} \ \text{ifcond } g\langle\langle\text{ifcond}\rangle\rangle \ g \\
\hline
\text{ConditionalEliminationStep } \text{conds} \ \text{stamps } g \ \text{ifcond } g'
\end{array}$$

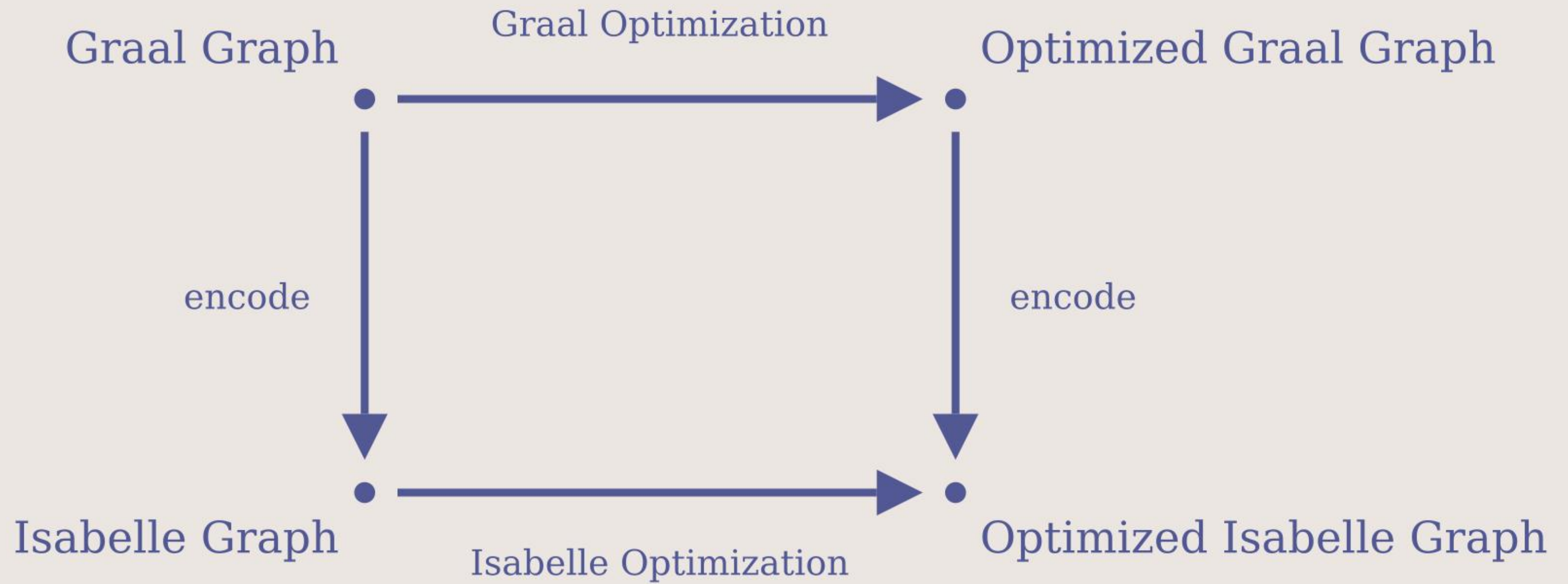
$$\begin{array}{c}
g\langle\langle\text{ifcond}\rangle\rangle = \text{IfNode } cid \ t \ f \\
\text{cond} = g\langle\langle cid \rangle\rangle \quad \exists c \in \text{conds. } g \vdash c \ \& \ \text{cond} \mapsto \text{KnownFalse} \\
g' = \text{constantCondition } \text{False} \ \text{ifcond } g\langle\langle\text{ifcond}\rangle\rangle \ g \\
\hline
\text{ConditionalEliminationStep } \text{conds} \ \text{stamps } g \ \text{ifcond } g'
\end{array}$$

$$\begin{array}{c}
g\langle\langle\text{ifcond}\rangle\rangle = \text{IfNode } cid \ t \ f \\
\text{cond} = g\langle\langle cid \rangle\rangle \quad \text{tryFold } g\langle\langle cid \rangle\rangle \ \text{stamps } \text{KnownTrue} \\
g' = \text{constantCondition } \text{True} \ \text{ifcond } g\langle\langle\text{ifcond}\rangle\rangle \ g \\
\hline
\text{ConditionalEliminationStep } \text{conds} \ \text{stamps } g \ \text{ifcond } g'
\end{array}$$

$$\begin{array}{c}
g\langle\langle\text{ifcond}\rangle\rangle = \text{IfNode } cid \ t \ f \\
\text{cond} = g\langle\langle cid \rangle\rangle \quad \text{tryFold } g\langle\langle cid \rangle\rangle \ \text{stamps } \text{KnownFalse} \\
g' = \text{constantCondition } \text{False} \ \text{ifcond } g\langle\langle\text{ifcond}\rangle\rangle \ g \\
\hline
\text{ConditionalEliminationStep } \text{conds} \ \text{stamps } g \ \text{ifcond } g'
\end{array}$$

Validating Faithful Semantics of an Existing Compiler





What's Next?

```
boolean associative = op.isAssociative();
if (associative) {
    if (forX instanceof SubNode) {
        SubNode sub = (SubNode) forX;
        if (sub.getY() == forY) {
            // (a - b) + b
            return sub.getX();
        }
    }
}
```

```

boolean associative = op.isAssociative();
if (associative) {
    if (forX instanceof SubNode) {
        SubNode sub = (SubNode) forX;
        if (sub.getY() == forY) {
            // (a - b) + b
            return sub.getX();
        }
    }
}

```

$$(\text{BinaryExpr BinAdd } (\text{BinaryExpr BinSub } a \ b) \ b) \mapsto a$$

```

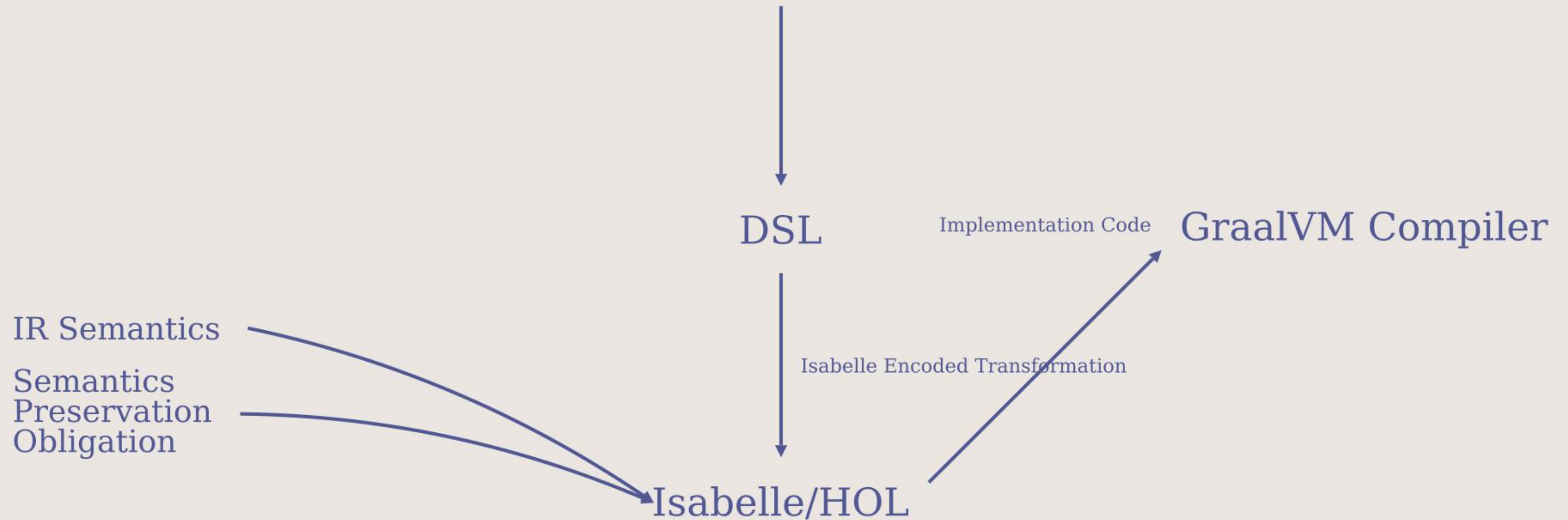
boolean associative = op.isAssociative();
if (associative) {
    if (forX instanceof SubNode) {
        SubNode sub = (SubNode) forX;
        if (sub.getY() == forY) {
            // (a - b) + b
            return sub.getX();
        }
    }
}

```

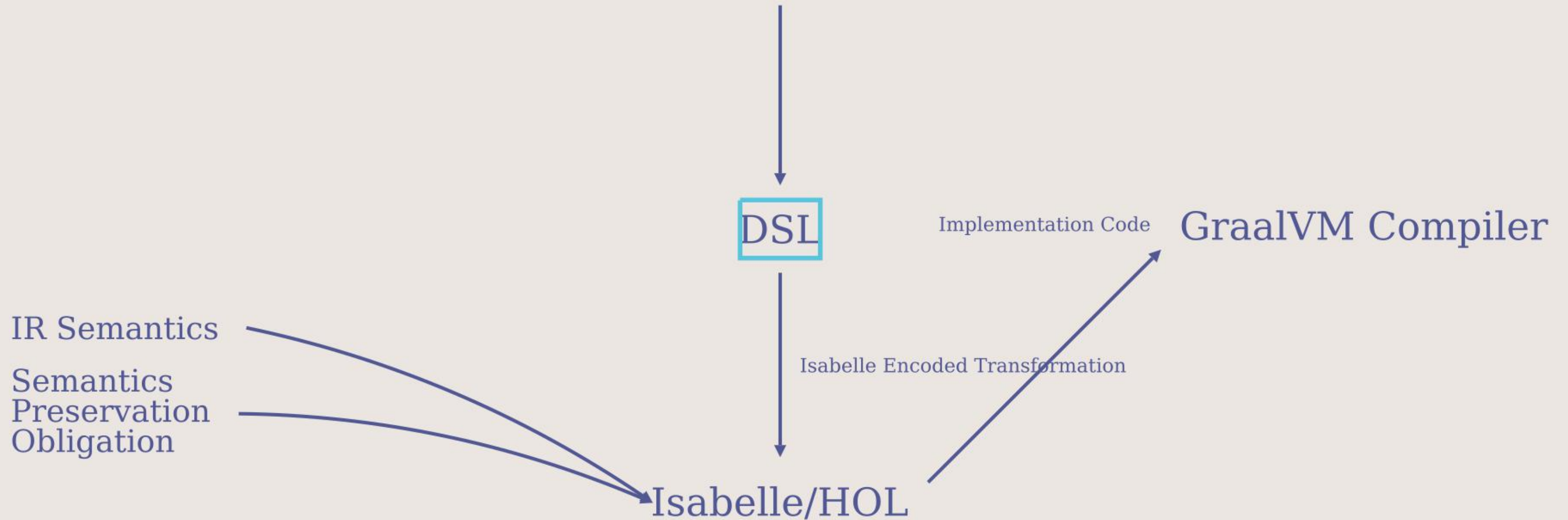
$$(\text{BinaryExpr BinAdd } (\text{BinaryExpr BinSub } a \ b) \ b) \mapsto a$$

$$(a - b) + b \mapsto a$$

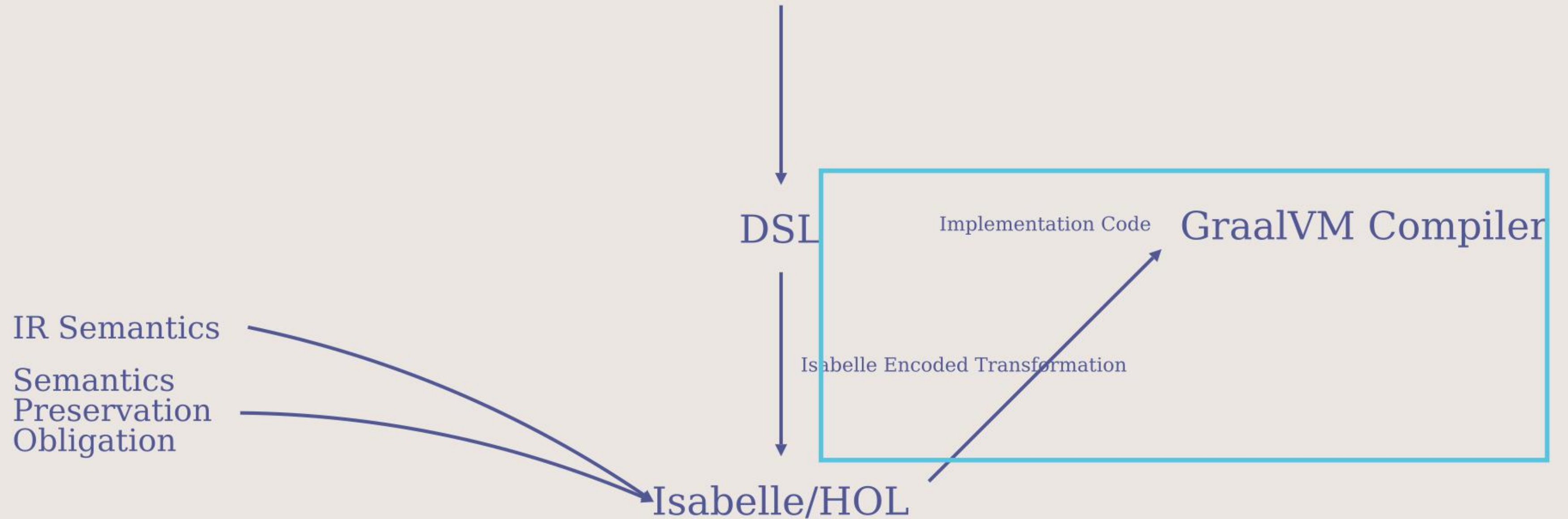
Optimization Transformation Definitions

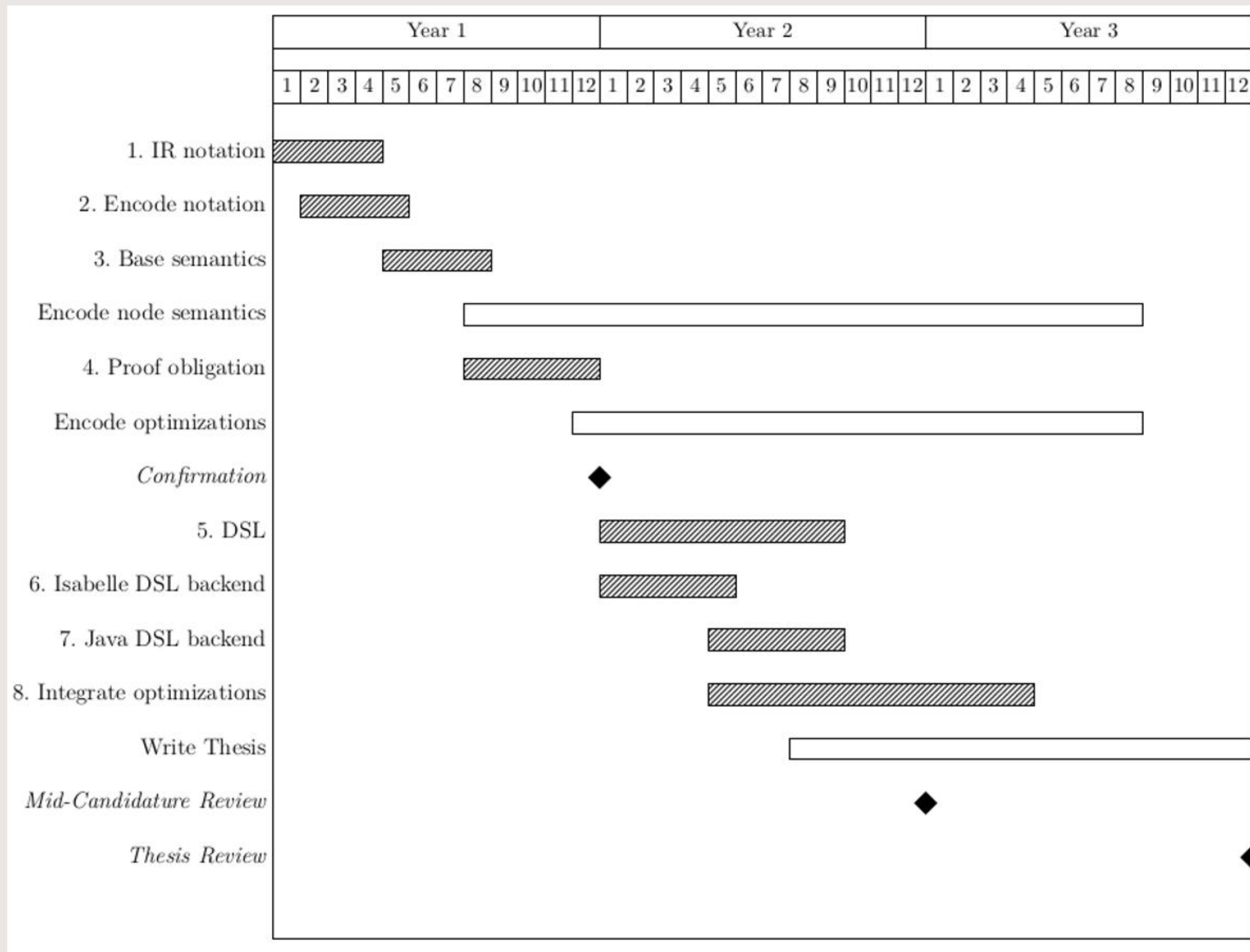


Optimization Transformation Definitions

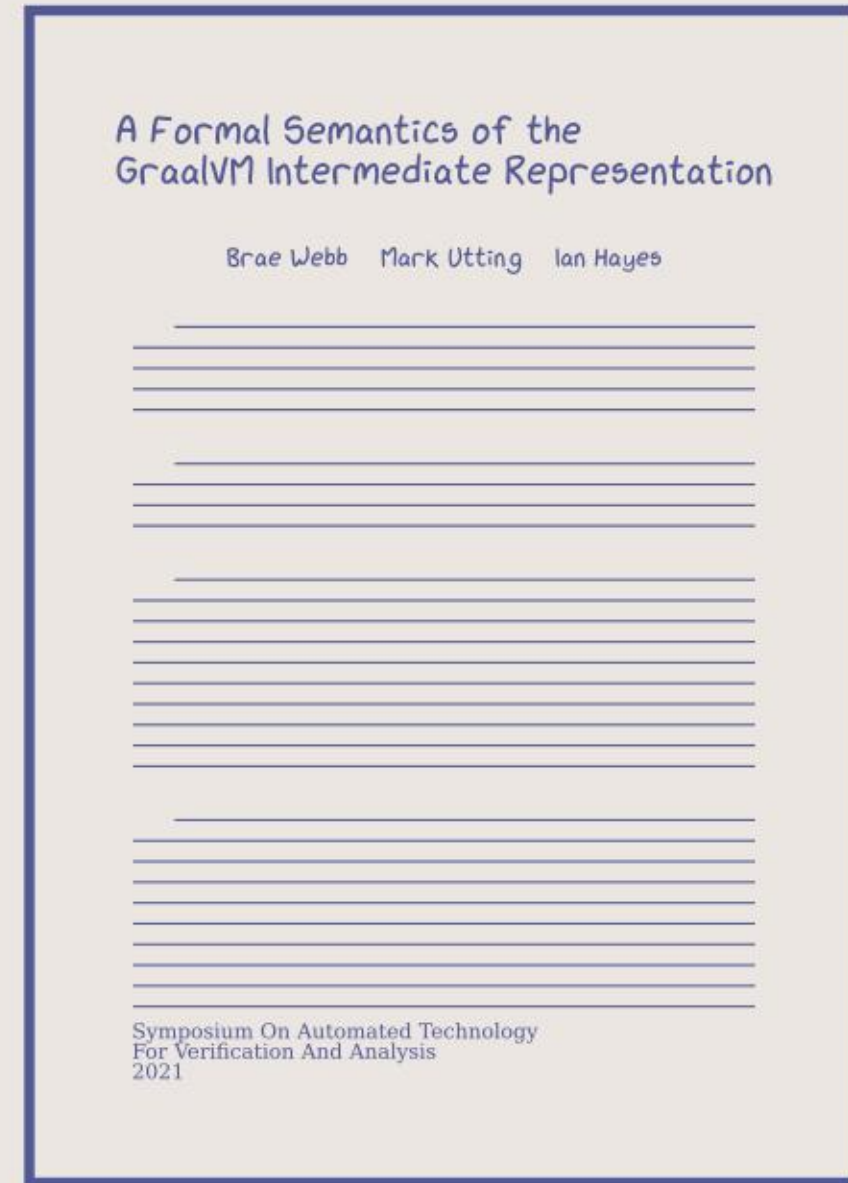
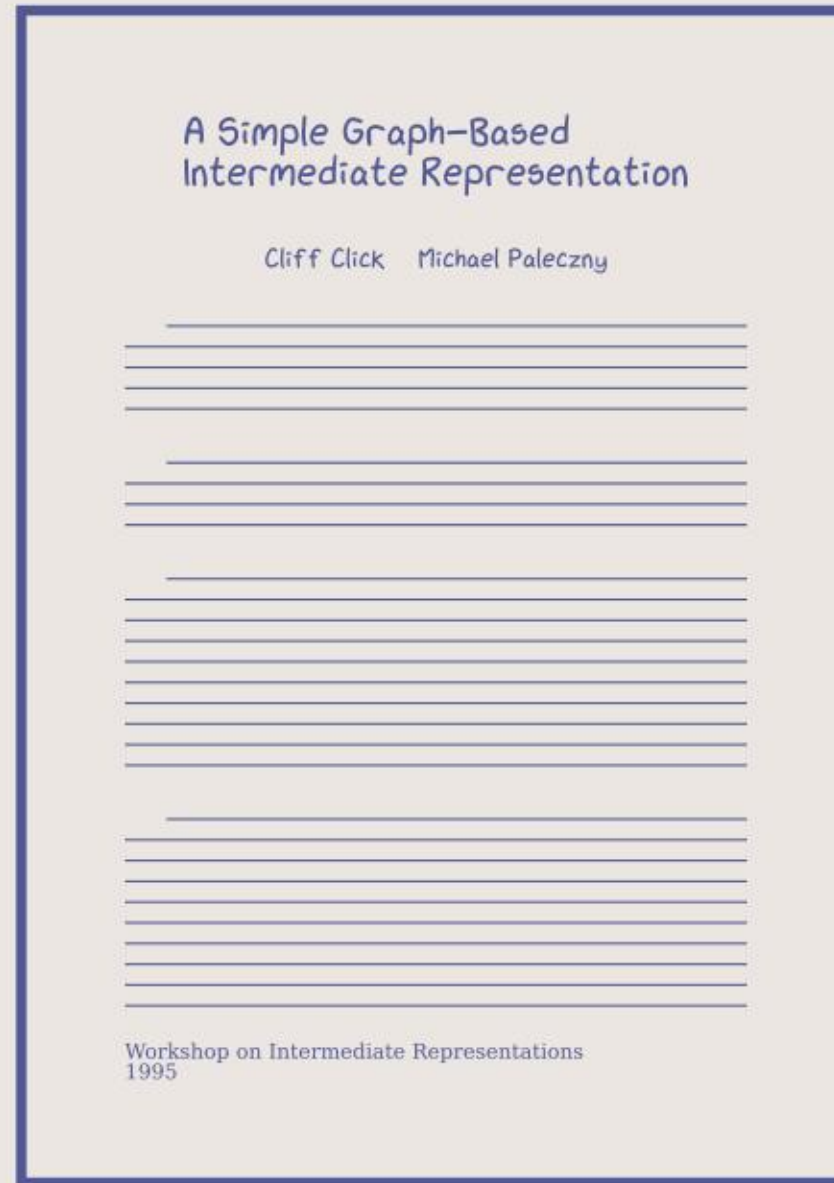
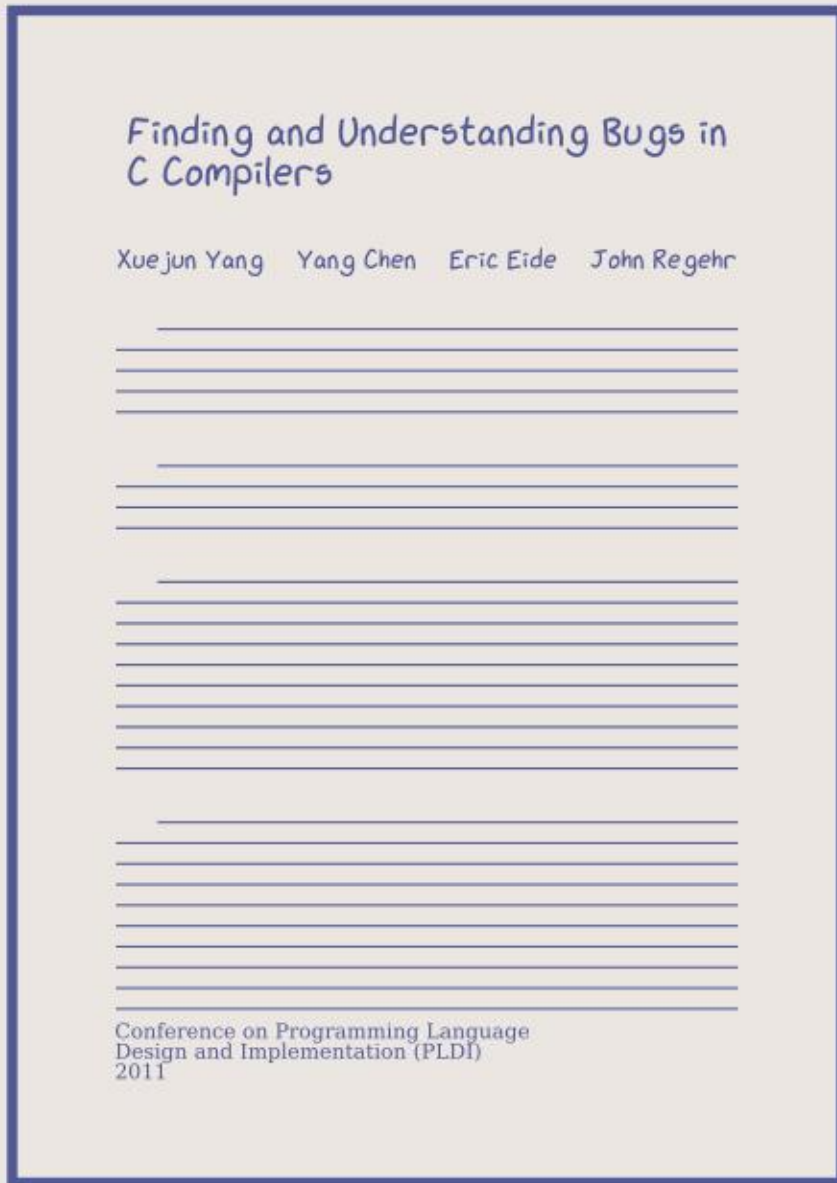


Optimization Transformation Definitions





Thank You!



<https://phd.brae.dev>

